SHORT PRACTICAL RULES

FOR

COMMERCIAL CALCULATIONS.

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COMMERCIAL CALCULATIONS,

INCLUDING

DIVISION SIMPLIFIED AND ABBREVIATED,

AND

SHORT METHODS FOR MULTIPLICATION: ORIGINAL AND INGENIOUS METHODS IN CANCELLATION, THE RULE OF THREE, PERCENTAGE, INTEREST AND DISCOUNT, AND A SIMPLE METHOD FOR AVERAGING ACCOUNTS. TO WHICH

IS ADDED AN

EXPOSITION OF PROFIT AND LOSS, DIVISION INTO PROPORTIONAL PARTS,

PARTNERSHIP AND BANKRUPTCY,

INVOLUTION, COMPOUND INTEREST, ANNUITIES, SINKING FUND, AND BOND COMPUTATIONS, STERLING, CHRONOLOGICAL CALCULATIONS. A SIMPLE METHOD FOR THE EXTRACTION OF THE CUBE ROOT,

AND

A SIMPLE RULE SHOWING HOW TO DISCHARGE A GIVEN DEBT IN SEVERAL EQUAL PAYMENTS, IN A GIVEN TIME, INCLUDING PRINCIPAL AND INTEREST, AT A GIVEN RATE PER CENT.

A

SIMPLE METHOD FOR ADDITION,

AND

PRACTICAL HINTS FOR BUILDERS, ETC.

BY

PATRICK MURPHY.

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PREFACE.

A trivial matter having led the author to make some investigations in the fundamental rules of Arithmetic, particularly Division, and having, in the course of these investigations, discovered some novel methods for simplifying and abbreviating Long Division, he has concluded to present them in this form to the public, flattering himself that they will be found, on perusal, both interesting and instructive.

In treating the matter, the author assumes that the reader has a knowledge of arithmetical notation and numeration, the simple rules, and the principles of Division as given in most Arithmetics of the present day, and which will be essential to a proper understanding of the subject; hence, the explanation of technical terms, definitions, etc., unless when necessary, will be found omitted.

Division, we are aware, could not be changed without making, at the same time, a corresponding change in Multiplication — one being the reverse of the other — hence, some extraordinary results will be found in the contractions in Multiplication, to which attention is invited.

The method for Addition will be found advantageous whenever the person engaged in adding is liable to interruptions.

The rule for finding the weekly day, which forms the closing chapter of this little work, is quite simple, requiring no arithmetical process beyond Division for its solution, and is easily remembered, as it requires neither monthly nor centennial ratio to be added. The rule is original so far as known to the

AUTHOR.

ALBANY, *January*, 1886.

NEW EDITION

REVISED, ENLARGED AND IMPROVED.

In the preparation of this work, our first attempt at authorship, Division Simplified, etc., has formed the groundwork of construction. The present edition contains the principal feature of the original work, the *Vertical Line*, by the use of which, most extraordinary results in arithmetical calculations are obtained, and problems solved much more readily than by the usual methods given in text-books.

In the new work, we have endeavored to make the use of the line more simple and clear, explaining the most important principles connected with it, and giving numerous examples and illustrations.

Original and Ingenious Methods will be found in Cancellation, The Rule of Three, Percentage, Interest and Discount, Averaging Accounts, Profit and Loss, Division into Proportionate Parts, Partnership, Bankruptcy, and Practical Hints for Builders, which are given in the present work.

In presenting this edition, the Author trusts that, by the improvements now introduced, it will be rendered worthy, in a much greater degree, of a continuance of the very favorable reception which it has already experienced from a kind public.

THE AUTHOR TO THE READER.

Before attempting the simple and novel methods given in Cancellation, the Rule of Three, Interest, Discount, etc., *a thorough knowledge of our methods for Division and Multiplication will be necessary*, as without the latter, the simplicity and novelty of the former can neither be understood nor appreciated.

January, 1889.

IMPROVED EDITION.

Some very important improvements have been made in this edition, and fifteen pages of new matter added, which, from long experience, the Author knows will be found useful and practical. To particularize all that has been done would be tedious and unnecessary: suffice it to say that, in the present edition, a more comprehensive and lucid explanation of our Simple Method for Averaging Accounts is given; Short Rules for converting British Sterling to American Currency, and for computing Interest on British Sterling; Hints on Interest, and Other Short Methods, together with a Simple Rule for finding the face of a note, the proceeds being given. This, we will venture to say, is the first time the rule has appeared in print, and will be found of great practical utility.

The *reasons* of the rules and operations (a part of arithmetical science too generally neglected both in treatises on the subject, and in teaching), are fully explained by simple and easy illustrations and examples; and it is hoped that the subjects will thus be rendered intelligible and attractive to the reader.

July, 1895.

ENLARGED NEW EDITION.

Twenty pages of useful and practical matter, carefully prepared, have been added in this edition.

The article on Interest Simplified cannot fail to interest the student. The Short methods for finding Interest on Daily Balances, and for changing Commercial into Exact Interest, and the reverse; also Short Methods on Tonnage, both Net and Gross; and the Short Methods on Trade Discounts, together with numerous examples and illustrations, and the *reasons* for those Short Methods, will be found worthy the attention of the reader.

July, 1900.

We have added to the present edition a Review on Interest, with illustrations and examples, the most complete, perhaps, ever given on this subject. We have also made important changes in the body of the work, and added fourteen pages of Hints and Helps for the Student, which will be found both useful and interesting.

August, 1904.

REVISED, ENLARGED AND IMPROVED EDITION.

Several changes of a very important nature have been made in the present edition. Twenty-one pages have been thrown out, and new matter of a more useful and business character substituted. Besides this, Twenty-five *new* pages have been added, consisting chiefly of very simple methods for multiplying together numbers of *two, three, four, five and more figures*, whether whole or fractional, with copious *examples, illustrations and reasons.* These methods, so far as known to the Author, are entirely *original*, and will be found exceedingly simple and practical.

August, 1906.

Some things of minor importance, contained in former editions, have been omitted in this edition, and the space thus gained has been filled by the insertion of valuable rules, which cannot fail to be interesting to the student.

July, 1908.

LATEST EDITION, ENLARGED AND IMPROVED.

To this, the latest and best edition of the work, we have added a Chapter on Involution; Compound Interest, including a Table showing the Amount of $1, or £1 sterling from 2% to 10%, for any number of years from 1 to 35; and a Simple Rule for the Computation of Bonds; and for Sinking Funds; also a Rule to Ascertain the Amount necessary to Discharge a Given Debt in Several Equal Payments in a given time, including both Principal and Interest, at a given rate per cent.

In connection with this rule, we have given a Table showing the Amount necessary to discharge the Debt of $1, in equal payments, from 2 to 21 years, thereby facilitating the solution of such problems.

Much labor and care have been given to the construction of this Table and, so far as known to the Author, it is the first of its kind given in an Arithmetical work.

We have also given a Short Rule for the Computation of Paper; and a Simple Rule for the Extraction of the Cube Root.

The rules are Tersely expressed; the Problems are all solved and fully illustrated, so that the Student will have no difficulty in mastering the different subjects.

January, 1912.

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APPENDIX.

DIVISION

SIMPLIFIED AND ABBREVIATED.

General Principles.

There are certain general principles of Division, a knowledge of which is essential to a proper understanding of the simplified methods given in this work. The following are some of the most important:

I. Since the quotient in Division is the result obtained by dividing the dividend by the divisor, it is evident that the *value of the quotient* depends upon the relative values of the dividend and divisor. Hence,

Any change in the value of either dividend or divisor must produce a change in the value of the quotient; but if a similar change be made in both dividend and divisor, at the same time, the quotient undergoes no change.

II. If a number be *added* to the divisor, the dividend must be *increased* by the product of the quotient and the number so added, in order that the quotient may remain the same. Thus,

$$84 \div 10 = 8 \ldots 4$$

Now, if any number, say 2, be added to the divisor, 10, and we desire to divide by 12, without changing the quotient of 10, we increase 84 by 2 times the quotient 8, or by 16; then, dividing the sum, we have $100 \div 12 = 8 \ldots 4$, the same as was obtained by dividing 84 by 10.

And if the dividend be *not* thus increased, the quotient will be *diminished* by the result obtained by dividing the product of the quotient and the number added by the new divisor. Thus,

$$84 \div 10 = 8 \ldots 4$$

Now, if we divide by 12, without increasing 84, we have,

$$84 \div 12 = 7$$

the same as if 2 times 8 were divided by 12, and the result subtracted from the quotient of 10. Thus,

$$\begin{array}{rl} & 84 \div 10 = 8 \ldots 4, \text{ less 2 times 8, or} \\ \text{16 divided by 12:} & 16 \div 12 = \underline{1 \ldots 4} \\ & \qquad\qquad\;\; 7 \end{array}$$

NOTES. — 1. When the divisor is contained in the dividend without a remainder, the division is exact.

2. When the division is not exact, a part of the dividend is left, this is called the *remainder* and must be less than the divisor.

3. The remainder is always of the same name or kind as the dividend, being a part of it.

The division of one number by another is denoted in several ways. Thus, either of the expressions, $84 \div 10 = 8 \ldots 4$; $\frac{84}{10} = 8 \ldots 4$; or $10\overline{)84}$ means that 84 is divided by 10, that
$\quad\;\; 8 \ldots 4$

the quotient is 8, and the remainder 4, fully expressed $8\frac{4}{10}$.

The quotient, also, may be expressed in several ways. Thus, $8\frac{4}{10}$, $8^4/_{10}$, 8.4, or, if we substitute a vertical line for the point in the last expression, it may be expressed thus, 8 | 4, showing that, to divide a number by 10, we simply cut off the unit figure of the dividend for remainder.

RULE I.

To divide by a number expressed by 1, *with a cipher or ciphers annexed: Cut off from the right of the divi-*

dend, by a vertical line, as many figures for remainder, as there are ciphers in the divisor.

EXAM. 1. Divide 475891 by 1000.

$$475|891$$

Here, there are three ciphers in the divisor, and we cut off from the right of the dividend, three figures, 891, for remainder, and 475 is the quotient; fully expressed, $475\frac{891}{1000}$.

Now, let a number, say 7, be added to the divisor, 1000, and let the given dividend, 475891, be divided by the sum, or new divisor, 1007, according to principle II. Thus:

$$\begin{array}{r|l} 475 & 891 \\ 3 & 304 \\ \hline 472 & 587 \end{array}$$

Dividing first by 1000, we get 475 for quotient, and 891 for remainder.

To finish the division, now, according to the principle referred to, we multiply the quotient, 475, by 7, the figure added, and divide the product, 3325, by the new divisor, 1007, as shown in the margin, getting 3 for quotient, and 304 for remainder. Subtracting this result from 475|891, found above, we get the true quotient, 472, and the true remainder, 587; fully expressed, $472\frac{587}{1007}$.

$$\begin{array}{r} 1007)3325(3 \\ 3021 \\ \hline 304 \end{array}$$

The required quotient for 1007, *however, may be more readily obtained as follows:*

$$\begin{array}{r|l} 475 & 891 \div 1007 \\ 3 & 325 \\ \hline 472 & 566 \\ & 21 \\ \cline{2-2} & 587 \end{array}$$

First, divide by 1000; the quotient is 475, and the remainder, 891. Then, extending the vertical line to a suitable length, multiply the quotient, 475, by the excess, or added figure 7, and set the product, 3325, so that its unit figure will be under the unit figure of the remainder, 891, and *subtract* the said product from 475|891.

Next, multiply 3, that part of the product (now partial quo-

tient), to the left of the line, by 7, also; set the result, 21, under the remainder, 566, and *add* both; the sum, 587, is the true remainder, and 472, to the left of the line, is the quotient.

The *reason* of the last process will be understood by comparing it with that which immediately precedes, in connection with the well-known principles: (*a*) *the greater the divisor, the dividend remaining unchanged, the less will be the value of the quotient; and* (*b*) *the less the divisor the greater the quotient.*

The extended vertical line, observe, divides 3325, as well as 475891, by 1000. Now, the product 3325 being divided by 1000 in the last process, while it ought to have been divided by 1007, as in the first (Prin. II), the result, 3|325, is too large (Prin. b); and subtracting this result, which is too large, from 475|891, leaves too little; consequently, we must *add* the excess taken away.

This excess, observe, is the difference between 3|325, in the last process, and 3|304, the true result taken away in the first process, or 21, which is 7 times 3, the partial quotient, or that part of the product, 3|325, to the left of the vertical line.

Or the reason may be explained thus: The true result to be subtracted is 3|304; we have subtracted 3|325; the difference is 21. Having taken away 21 too many we restore the same by addition.

Before giving the rule for this method of division, the following *four* examples are necessary, as they still further illustrate our method and explain some changes which will frequently occur when making use of this method:

EXAM. 2. Divide 3487286 by 1006.

$$\begin{array}{r|r}
3487 & 286 \\
20 & 922 \\
\hline
3466 & 364 \\
 & 126 \\
\cline{2-2}
 & 490
\end{array}$$

In this, we first divide by 1000, as in the previous example, getting 3487 for quotient and 286 for remainder. Then, multiplying 3487 by 6, the product, 20922, is set under the first result and subtracted; the difference is 3466|364. In the subtraction 1 is carried, which makes 21 to the left of the vertical line, and in multiplying next by 6, we multiply *not* 20, but 21 (20 plus the 1 carried), setting the product, 126, under the remainder, 364, and adding; the true remainder is 490, and the true quotient 3466.

NOTE. — The number carried from the remainders to the quotients forms a part of the quotient to which it has been carried, and must evidently be multiplied in with it.

EXAM. 3. Divide 348728654 by 1010.

$$\begin{array}{r|r}
348728 & 654 \\
3487 & 280 \\
\hline
5241 & 374 \\
34 & 870 \\
\hline
76 & 244 \\
 & 350 \\
\hline
345275 & 894 \\
 & 10 \\
\cline{2-2}
 & 904
\end{array}$$

Here, we first divide by 1000 as in the other examples, cutting off 654 for remainder. Then, multiplying the quotient, 348728,

by the excess, 10 (to multiply by 10, we simply annex a cipher and copy the figures), the product, 3487280, is set as in the preceding examples, and subtracted (omitting the two outside figures, 34, for the present, as they undergo no change by the subtraction).

Next, multiplying that part of the product, 3487, to the left of the line, by 10, also, and setting the product, 34870, as before, it is added (omitting the figures 52 for the present).

In the addition 1 is carried to 34, making 35, which is now multiplied by 10, the product 350 set as before and subtracted. The figures 3452 are now brought down, giving 345275 for quotient.

In subtracting 350 from the result above it, 1 was carried and taken from 76, giving 75; and if actually expressed, said 1 would be set under 76. It is evident, therefore, that such a figure is a partial quotient, the same as 3487 and 34, and must be treated accordingly; hence, 1 is multiplied by 10, also, and the product 10 added to 894, giving 904 for the true remainder.

EXAM. 4. Divide 3509615 by 1012.

$$\begin{array}{r|l} 3509 & 615 \\ 42 & 108 \\ \hline 3467 & 507 \\ & 504 \\ \hline \multicolumn{2}{r}{1011} \end{array}$$

In this we divide by 1000, as in the other examples, and multiply by the excess, 12, getting 3467 for quotient and 1011 for remainder.

In adding the remainders 504 and 507, it would appear that we ought to have carried 1 to the quotient's place, multiplied said 1 by 12, as in the preceding example, and subtracted; but we observe that the sum, 1011, is *less* than the divisor 1012, and, therefore, 1011 is the final remainder.

EXAM. 5. Divide 7423520 by 1040.

$$
\begin{array}{r|l}
7423 & 52|0 \div 104|0 \\
296 & 92 \\
\hline
126 & 60 \\
11 & 88 \\
\hline
7138 & 48 \\
 & 48 \\
\end{array}
$$

In this, we cut off the cipher from the right of the divisor, 1040, and also the cipher from the right of the dividend. Then, dividing 742352, the remaining part of the dividend, by 104, the remaining part of the divisor, as in the preceding examples, we get 7138 for quotient, and there is no remainder.

In multiplying 296 and 11 by 4, the excess in 104, it must be borne in mind that 1 has been carried from the remainder 92, in subtracting, and 1 also from the remainder 88, in adding, and that we have multiplied 297 in the one case, and 12 in the other, by 4, getting 1188 and 48.

NOTE.—It need scarcely be remarked that the numbers to the right of the vertical line are remainders, and those to the left quotients; that is, partial remainders and partial quotients, till the final quotient and remainder are found.

RULE II.

From the foregoing examples and illustrations we derive the following:

RULE. *To divide by a number which is a little in excess of* 100, 1000, 10000, etc.:

I. *Divide, first, by* 100, 1000, 10000, *etc., as the case may be, using a vertical line to cut off the remainder, as pointed out in Rule I.* (Call 100, 1000, etc., in such cases, the approximate divisor.)

II. *Extend the vertical line to a suitable length, and multiply the figures to the left of said line by the excess figure of the divisor, setting*

the product under the first result, so that units will be under units, etc., *and subtract.*

III. *If a figure or figures of the said product extend to the left of the vertical line, multiply such figure or figures, also, by the excess figure of the divisor, and include in the multiplication, the figure carried, if any, from the remainder to the quotient, in the subtraction; set the result to the right under the difference already found, and add; and so on, as long as possible, subtracting and adding alternately, and including in each multiplication the figure carried from the remainders to the quotients in subtracting and adding.*

NOTE.—When the last product figure, or carried figure, to the left of the line, is brought to the right by multiplication, the division is completed, the quotient being to the left, and the remainder to the right, of the vertical line.

The *reason* why we subtract and add *alternately* will be understood from the following, in connection with the principles of Division already laid down.

By referring to example 3, page 13, it will be seen that, instead of dividing 3487280 (the product of the quotient and the figure added, or the excess 10) by 1010, the *real* divisor, we have divided by the *approximate* 1000, thereby getting too large a quotient; subtracting that which is too large leaves too little, as has been already explained; consequently, something must be added to rectify.

The true result to be added would be that found by dividing the product of 3487 and 10, or 34870, by the real divisor, 1010. Instead of dividing 34870 by 1010, however, we prefer to divide it by the approximate divisor, 1000, getting 34|870, which is added.

The result thus added being too large (having divided by a smaller divisor than the real one), we obtain too much by the addition, and, therefore, we must subtract next; and so on, till the parts of all the products to the left of the line are brought to the right by multiplication; whence, the reason of multiplying by the excess figure of the divisor, and setting the products as

directed by the rule is evident, and the reason for subtracting and adding alternately is shown.

Or the reason might be expressed in general terms as follows: Dividing in each case by a quantity which is too small, gives a result in each case too large. Subtracting too large a quantity leaves too little — adding too large a quantity gives too much — we must subtract and add till the proper correction is made.

NOTE.— Each new correction becomes less than the previous one, till it finally disappears.

TO FIND THE DECIMAL.

Should it be desirable to continue the division into decimals, in example 8, the process, according to the usual method for Long Division, would be to annex a suitable number of ciphers to the remainder, 904, and keep on dividing by 1010 till the required number of decimals is found. Thus, suppose we required three decimals in said example:

```
1010)904000(.895
     8080
     ----
      9600
      9090
      ----
       5100
       5050
       ----
```

Annexing three ciphers and continuing the division we get .895+

SIMPLE METHOD OF FINDING THE DECIMAL.

To find the decimal from the remainder by the simplified process: Annex, or conceive to be annexed, as many ciphers to the remainder as there are decimals required, and continue the division as in the first part of the process.

Thus, taking the remainder, 904, in example 3, again, we conceive three ciphers annexed, and continuing the process, as in the first part of the example, we get .895+ as before.

$$\begin{array}{r|l}
904 & \ldots \div 1010 \\
9 & 040 \\
\hline
894 & 960 \\
 & 100 \\
\hline
.895 &
\end{array}$$

NOTE.—This will be found more fully explained in a subsequent part of the work.

GENERAL PRINCIPLES.

III. If a number be taken from the divisor, the dividend must be *diminished* by the product of the quotient and the number subtracted, in order that the quotient may not be changed. Thus,

$$84 \div 12 = 7$$

Now, if 2 be taken from the divisor, 12, and we desire to divide by 10, without changing the quotient of 12, we subtract from 84, the dividend, 2 times the quotient 7, or 14; then dividing the difference, 70 (84 — 14), by 10 (12 — 2), we have:

$$70 \div 10 = 7$$

the same as: $84 \div 12 = 7$

And if the dividend be *not* thus diminished, the quotient will be *increased* by the result obtained by dividing the product of the quotient and the number subtracted, by the new divisor. Thus,

$$84 \div 12 = 7$$

Now, if we divide by 10, without diminishing 84, we have:

$$84 \div 10 = 8 \ldots 4$$

the same as if 2 times 7, or 14, were divided by the new divisor, 10, and the result added to the quotient of 12. Thus,

$$\begin{array}{rl}
 & 84 \div 12 = 7\text{, plus 2 times 7, or} \\
\text{14, divided by 10:} & 14 \div 10 = \underline{1 \ldots 4} \\
 & \phantom{14 \div 10 = {}} 8 \ldots 4
\end{array}$$

From a due consideration of the foregoing principles, the following illustrations will be readily understood:

If 9216487 be divided by 1000, as in example 1, Rule I, the result will stand thus:

$$9216|487$$

Suppose, now, we *subtract* any number, say 2, from the divisor 1000, and divide the same dividend, 9216487 (without diminishing it) by the difference, 998, according to the foregoing general principles. Thus,

$$\begin{array}{r|l} 9216 & 487 \\ 18 & 468 \\ \hline 9234 & 955 \end{array}$$

Dividing first by 1000, the quotient is 9216, and the remainder 487. To finish the division, now, according to Prin. III, we multiply the quotient 9216 by 2, the figure subtracted from the divisor, or the difference between 1000 and 998, and divide the product. 18432, by the new divisor, 998, by Long Division, as in the margin, getting 18 for quotient and 468 for remainder. This quotient and remainder is now *added* to 9216|487 found above, and the division by 998 is completed, 9234 being the true quotient, and 955 the remainder.

$$\begin{array}{r} 998)18432(18 \\ \underline{998} \\ 8452 \\ \underline{7984} \\ 468 \end{array}$$

The required quotient for 998, *however, can be more simply found as follows:*

$$\begin{array}{r|l} 9216 & 487 \div 998 \\ 18 & 432 \\ & 36 \\ \hline 9234 & 955 \end{array}$$

Dividing first by 1000, the quotient is 9216, and the remainder, 487. Then, extending the vertical line to a suitable length, the quotient, or figures 9216, to the left of the line, is multiplied by 2, the difference between 1000 and 998, and the product, 18432, set so that its unit figure will be under the unit figure of the remainder, 487, and the other figures in the corresponding places. Then, 2 times 18 (to the left of the line), or 36, is set in proper position under the remainder, 432; the results are now added the same as in simple addition, giving 9234 for the true quotient, and 955 for remainder; fully expressed, $9234\frac{955}{998}$.

The *reason* of this simple process will be understood by comparing it with that which immediately precedes, in connection with the principle: *the greater the divisor, the dividend not being changed, the less the quotient.*

Dividing first by 1000, which is greater than the real divisor, 998, the quotient obtained is too small and requires an addition. The true result to be added, it will be remembered, is the quotient, 18, and the remainder, 468, obtained by Long Division, by dividing 2 times 9216, or 18432 by 998, as shown in the process immediately preceding.

Instead of dividing 18432 by 998, however, as in the said process, we prefer, here, to divide it by 1000. The extended vertical line, observe, performs such division by merely setting the product 18|432, as shown in the margin.

Since 18432 has now been divided by 1000, while it ought to have been divided by 998, the result found, or 18|432, is likewise too small, and must have an addition also. The result required to be added is evidently the difference between the *true* result, 18|468, found in the first process, and 18|432, found in the second. The difference is 36, or 2 times 18, that part of the product, 18|432, to the left of the vertical line.

The reason of the simplified process might be expressed in gen-

eral terms as follows: Dividing by a quantity which is too large, in every case, gives a result which is too small, and we have to keep on adding till the proper correction is made.

TO FIND THE DECIMAL.

To find the decimal for the remainder, 955, true, say to four places, the usual method would stand thus:

$$\begin{array}{l}
998)9550(.9569+ \\
\underline{8982} \\
5680 \\
\underline{4990} \\
6900 \\
\underline{5988} \\
9120 \\
\underline{8982}
\end{array}$$

By the simplified process, we simply annex, or conceive to be annexed, four ciphers and continue the division as in the other part of the work, thus:

$$\begin{array}{r|l}
9550 & \ldots \\
19 & 100 \\
 & 38 \\
\hline
.9569 &
\end{array}$$

Conceiving four ciphers annexed (three of them represented by dots), 2 times 9550, or 19100; then 2 times 19 are placed in proper position, as in the first part of the process; then, adding the results (rejecting that part of the decimal to the right of the line, being less than 5), we have .9569, as found by the long process.

Before giving the rule for this interesting method of division, the *three* following examples are necessary to

still further illustrate our method, and explain a few simple changes which will frequently occur when making use of this method:

EXAM. 6. Divide 4789365 by 993.

$$\begin{array}{r|r} 4789 & 365 \\ 33 & 523 \\ & 231 \\ & 7 \\ \hline 4823 & 126 \end{array}$$

Dividing first by 1000, in this, the quotient is 4789, and the remainder, 365. Then, multiplying 4789, the partial quotient to the left of the line, by 7 (the difference between 1000 and 993), the product, 33523, is set in proper position, as has been explained in the previous illustration. Next, that part of the product (now partial quotient), to the left of the line, or 33, is multiplied by 7, also, and the product, 231, set in position. All the figures to the left of the line having now been multiplied by 7, the next step is to add the several results. Before doing so, we make a short mental examination of the partial remainders, or numbers on the right of the line, to ascertain whether any thing is to be carried to the figures on the left; and we find 1 is to be carried to 33, making 34, which ought to have been multiplied by 7, instead of 33, giving 238 as the true result to be added instead of 231. The same result is obtained, observe, by allowing 231 to remain as it is, and add 7 times the 1 carried, or 7, placed in proper position. The quotient, then, is 4823, and the remainder 126, fully expressed, $4823\frac{126}{993}$.

EXAM. 7. Divide 641458207 by 9930.

64145	820|7	÷ 993|0
449	015	
3	143	
	21	
64597	9997	
1	9930	
64598	0067	

In this we first cut off the cipher from the right of the divisor, and also one figure, 7, from the dividend, which will be the last figure of the remainder. Then, dividing the remaining part of the dividend by 993, as in the last example, we get 64597 for quotient, and 999 for remainder. The 7 cut from the dividend is now brought down, making the remainder 9997; this remainder, being greater than the divisor, 9930, contains said divisor once more, 1 is added to 64597, giving 64598 for the true quotient and 67 is the remainder.

EXAM. 8. Divide 478498963 by 9991.

47849	8963	÷ 9991
43	0641	
	387	
47892	9991	

Here, we divide first by 10000, getting 47849 for quotient and 8963 for remainder. Then, multiplying the figures to the left of the vertical line by 9 (the difference between 10000 and 9991) and adding, as in the other examples, we have 47892 for quotient and 9991 for remainder. The remainder, being equal to the divisor, contains it once, and there is no remainder, but 1 is added to the quotient, giving 47893 for the true quotient.

RULE III.

From the foregoing examples and illustrations we have the following:

RULE. *To divide by a number which is a little less than* 100, 1000, 10000, *etc.:*

I. *Divide first by* 100, 1000, 10000, *etc., as the case may be, using a vertical line to separate the quotient from the remainder.* (Call 100, 1000, etc., in such cases the *approximate* divisor.)

II. *Extend the vertical line to a suitable length and multiply the figures to the left of the line by the difference between the approximate and the true divisor* (this difference is called the complement), *setting the product under the first result, so that units will be under units, etc.*

III. *If a part of the said product extend to the left of the line, multiply such part, also, by the complement, setting the product as before; and so on till the figures on the left of the line are exhausted.*

IV. *Add the several results; the true quotient will be on the left, and the remainder on the right, of the vertical line (not forgetting to multiply the figure carried, if any, from the remainders to the quotients, setting the product in proper position and adding it, as shown in example* 6).

Arithmeticians have given a simple rule for dividing by 100, 1000, etc. (Rule I of this work). *We have now established, in addition to said rule, two very important and equally simple rules*, namely: first, for such numbers as are a little in excess of 100, 1000, etc. (Rule II), and second, for such numbers as are somewhat less than 100, 1000, etc. (Rule III).

A knowledge of these two simple rules, together with the principles of Division, will now enable us to simplify Long Division to an extraordinary degree, and, as a mat

ter of course, problems in other branches of Arithmetic where Long Division has to be used. To do so, we draw very largely on the following simple principle:

Multiplying or dividing both dividend and divisor by the same number, does not change the value of the quotient. Thus,

$15 \div 3 = 5$, and 4 times 15 divided by 4 times 3, or $60 \div 12 = 5$

Again, $48 \div 12 = 4$, and the half of 48 divided by the half of 12, or $24 \div 6 = 4$

PROBLEMS.

From a due consideration of the foregoing rules, principles and illustrations, the student will readily understand the following problems:

EXAM. 1. If 26 building lots be sold for $90350, what is the average price of each lot?

To solve this problem, we simply divide the price by the number of lots.

Now, it is evident that if 4 times as many lots were sold for 4 times the money, the average price of each lot would still be the same; and as we can more readily divide now by 4 times 26, or 104 (Rule II), we divide 4 times the money, $361400 by 104, thus:

$90350 \div 26$

3614	00 ÷ 104
144	56
3469	44
5	80
3475	24
	24

Multiplying both dividend and divisor by 4, we take the products for a new dividend and new divisor.

Dividing first by 100, the quotient is 3614; this is next multiplied by 4, the excess, and the product, 14456, subtracted. Then 145 (144 plus the 1 carried in subtraction) is multiplied by 4, also, and the product, 580, added. Finally, 6 (5 plus the 1 carried in addition) is multiplied by 4, and the product, 24, subtracted; there is no remainder, \$3475 being the quotient, or the average price of each lot.

EXAM. 2. If 196 tons of iron cost \$7252, what is the price of a ton?

To solve this we divide 7252 by 196, the quotient is the price.

Now, it is evident that if half 196 tons be bought for half the money, the price per ton will be still the same; and as we can more readily divide by 98, half of 196, than by 196 itself, we divide half the money, or \$3626, by 98; thus:

$$\begin{array}{r|l} 36 & 26 \div 98 \\ & \underline{72} \\ & 98 \end{array}$$

Here we make use of Rule III. Dividing first by 100, the quotient is 36, and the remainder 26. Multiplying 36, the quotient, then, by 2 (100 — 98), the complement, and setting the product, 72, under the remainder, 26, we add. The remainder, here, being equal to the divisor, 98, evidently contains said divisor, once more, and hence the true quotient is \$37, the price per ton (see exam. 8).

EXAM. 3. If 334 suits of clothes cost \$12408.10, what is the price per suit?

Here we see that the divisor, 334, is nearly one-third of 1000,

and bearing in mind that we have now an easy method for dividing by the numbers at either side of 1000, etc., whether a little more, or a little less, we divide 3 times the price by 3 times the number of suits; thus:

```
   12408.10 ÷  334
 ------------  ----
 37|224.30  ÷ 1002
   | 74
   ---------
   15|030
     |  30
     -----
```

Multiplying both dividend and divisor by 3, we get $37224.30, and 1002. Dividing first by 1000, we get 37 for quotient, and 224.30 for remainder; we next multiply the quotient, 37, by 2, the excess figure of the divisor, and set the product, 74, under the unit figure of the dollars, 224. Then, subtracting 74 leaves a remainder of 150, evidently *dollars*, to be still divided by 1002. Now, besides $150 of a remainder, there is a remainder of 30 cents, also, or 15030 cents in all, to be divided by 1002. The division of the cents is performed the same as the dollars, first dividing by 1000, and subtracting 2 times 15, or 30. The result is then $37.15, the price per suit.

EXAM. 4. If 47 tons of iron cost $1703.75, what is the price per ton?

Here, we see at a glance that 2 times 47, or 94, is a simpler divisor than 47 itself, so we multiply the cost and the number of tons each by two, and operate with the products, thus:

```
34|07.50 ÷ 94
 2|04
  |12
--|------
36|23|50
     1|38
      | 6
   ---|--
    24|94
```

Here we make use of Rule III, dividing the dollars first by 100, and adding for the complement (6) we get \$36 for quotient, and a remainder of \$23 to be still divided by 94. Now, there is a remainder of 50 cents also, making 2350 cents in all for the remainder, to be divided by 94; the division is performed same as on the dollars, first dividing 2350 by 100, and adding for the complement (6), and we get 24 cents and 94 remaining; this remainder being equal to the divisor, its value is evidently 1, making 25 cents. The price is, then, \$36.25 per ton.

EXAM. 5. If 202 boxes of cigars cost \$875.40, what is the cost of 1 box?

$$\begin{array}{l}
\quad\underline{875.40} \div \underline{202} \\
4\,|\,37.70 \div 101 \\
\quad\;\underline{\;\;4\;\;} \\
\quad 33\,|\,70
\end{array}$$

Dividing the number of boxes and the cost; that is, the divisor and dividend, each by 2, the new divisor becomes 101, by which we divide according to Rule II, and we get \$4.33, the required cost.

In dividing the remainder, \$33.70, or 3370 cents, by 101, we first divide by 100; the next step is to multiply 33 by 1, set the product, 33, under 70 and subtract, but we see by inspection that in doing so the next remainder would be less than 5, or less than half a cent (the remainder would be .37), and without proceeding farther, we see that 33 is the correct number of cents; the answer is, therefore, \$4.33.

EXAM. 6. If 721 boxes of cigars cost \$3121.93, what is the cost of a single box?

$$\begin{array}{l}
\underline{3121.93} \div \underline{721} \\
4\,|\,45.99 \div 103 \\
\quad\,\underline{12} \\
\quad\,33
\end{array}$$

Here, we divide both dividend and divisor by 7, and the new divisor becomes 103.

After dividing $445 by 103, there is a remainder of $33.99, or 3399 cents; to divide 3399 by 103, we would first divide by 100, this would give 33 for quotient, and 99 for remainder; the next step would be to multiply 33 by 3, and subtract the product, 99, from the remainder, 99, which would give 0 for remainder; so we see without proceeding farther in the example, that $4.33 is the correct answer.

EXAM. 7. If 816 hats cost $3468, what is the cost of 1 hat?

$$\begin{array}{r|l}
\multicolumn{2}{r}{3468 \qquad \div \; 816} \\
\hline
4 & 33.50 \div \overline{102} \\
 & \;\;\underline{\;\;8} \\
\multicolumn{2}{c}{25}
\end{array}$$

In this we divide the terms by 8, and the new divisor becomes 102. The rest is now plain; the answer is $4.25.

REMARKS.

Remarks on Rule II.—The attentive student cannot fail to see, at this stage of the work, to what an extraordinary extent Long Division may now be simplified. Confining our remarks here to Rule II, without further reference, for the present, to that equally, if not more, important Rule III, which will be fully considered hereafter, we see that:

If we take the number 800, for instance, every number between it and 900, differing by 8, can be simplified when used as a divisor.

Thus, 808, 816, 824, 832, 840, 848, 856, 864, etc., divided by 8, will give 101, 102, 103, 104, 105, 106, 107, 108, etc., for new divisors, to which Rule II is applicable.

Again, if the numbers between 8000 and 9000, 80000 and 90000, etc., be taken, a similar relation will be found to exist.

Thus, 8008, 8016, 8024, 8032, 8040, 8056, etc., divided by 8, will give 1001, 1002, 1003, 1004, 1005, 1007, etc., for new divisors.

And the same is true with regard to the numbers from 200 to 300, 300 to 400, 400 to 500, 500 to 600, 600 to 700, etc.

Thus, 303, 306, 309, 312, 315, 318, etc., divided by 3, would give new divisors 101, 102, 103, 104, 105, 106, etc.; and the same may be said in regard to 3003, 3006, 3009, 3012, 3015, etc.; 30003, 30012, 30018.

And 909, 918, 927, 936, 945, 954, 963, 972, etc., divided by 9, give 101, 102, 103, 104, 105, 106, 107, 108, etc., for new divisors, to all of which Rule II is applicable; and so with the other series. Hence,

When the last figure or figures of the divisor are a multiple of the first figure or figures, the division can be simplified.

NOTE.— When one number is contained in another an exact number of times, the less is said to be a *measure* of the greater; and the greater is called a *multiple* of the less. Taking the divisor, 816, in the last example, for instance, 8, the first figure, is contained without remainder in 16, the two last figures. 8 is a measure of 16, and 16 is a multiple of 8.

And *when the first figure or figures of the divisor is a multiple of the last figure or figures*, Rule II is also applicable, as illustrated in the following:

EXAM. 8. If 2001 citizens of New York pay an annual tax of \$704352, what is the average tax to each?

$$\begin{array}{r|l} \multicolumn{2}{c}{704352 \div 2001} \\ \hline 352 & 176 \div 1000\tfrac{1}{2} \\ & 176 \\ \cline{2-2} \end{array}$$

To solve this problem, we divide the whole tax by the number of persons, thus:

Dividing the dividend and divisor, each, by 2, we have 352176 to be divided by $1000\frac{1}{2}$. Dividing by 1000, we get 352 for quotient, and 176 for remainder. The result thus found being too large, we multiply the quotient, 352, by the excess, $\frac{1}{2}$, and divide the product by 1000, and subtract the result. This result is found by simply taking half of 352, or 176, and setting it in proper position under the remainder, 176; the vertical line performing the division by 1000; and the answer is \$352, the average tax. Or thus:

$$
\begin{array}{r|l}
\multicolumn{2}{c}{7043520 \div \overline{20010}} \\
\hline
352 & 1760 \div \overline{10005} \\
 & 1760 \\
\cline{2-2}
\end{array}
$$

Annexing a cipher to both dividend and divisor; in other words, multiplying each by 10, the new divisor becomes 20010. Then, dividing by 2, we have 3521760 to be divided by 10005, according to Rule II, and we get \$352, as before.

EXAM. 9. If 804 building lots be sold for \$738476.34, what is the average price of each lot?

$$
\begin{array}{r|l}
7384 & 76.34 \;\div\; 804 \\
\hline
923 & 0954+ \;\div\; \overline{100\frac{1}{2}} \\
4 & 615 \\
\hline
918 & 480 \\
 & 25 \\
\cline{2-2}
 & 5054
\end{array}
$$

Here, we divide both dividend and divisor by 8, and the new divisor becomes $100\frac{1}{2}$, by which we divide as in example 8. (It being immaterial whether we divide first by 8 and next by 100, or

divide first by 100 and then by 8; we have here divided first by 100, thereby getting the position of the vertical line at once.) Multiplying the quotient, 923, by $\frac{1}{2}$, we get 461.5, and dividing this by 100 gives 4.615, which is subtracted. In subtracting, 1 is carried to 4, to the left of the line, making 5; this is multiplied by the excess, $\frac{1}{2}$, also, and the product, 2.5, divided by 100, giving .025 as result, which is added. The quotient is then **$918.50**, the average price of each lot.

Or thus:

$$
\begin{array}{r|l}
\multicolumn{2}{l}{73847634 \div 804\,..} \\
\hline
923 & 095|4 \div 1005|0 \\
4 & 615 \\
\hline
918 & 480 \\
 & \;\;25 \\
\hline
 & 5054
\end{array}
$$

Moving the decimal point two places to the right, in both dividend and divisor (this multiplies by 100), and then dividing each by 8, the new divisor becomes 10050, by which we divide, as in example 5, page 15, and we get $918.50, as before.

In applying the foregoing methods, *recourse may be had to any process whereby the divisor can be reduced to a simple one;* always bearing in mind that, *whatever change is made in the devisor a similar change is to be made in the dividend to preserve the relationship.*

By a simple divisor is meant 10, 100, 1000, 10000, etc.; or any number near to these, as 101, 1002, 10009, etc.; 91, 92, 994, 99997, etc.

EXAM. 10. Divide 369940704 by 77784.

Here, we divide the terms each by 7 and multiply the results by 9, getting 100008 for a simple divisor. The quotient is 4756.

$$
\begin{array}{r|l}
\multicolumn{2}{r}{369940704 \div \;\; 77784} \\
\hline
\multicolumn{2}{r}{52848672 \quad\;\; 11112} \\
\hline
4756 & 38048 \div 100008 \\
 & 38048 \\
\hline
\end{array}
$$

EXAM. 11. Divide 251076872 by 6668.

In this, we add one half the divisor and one half the dividend to each respectively and the divisor becomes 10002.

The quotient is 37654, the remainder, $\frac{10002}{10002}$, being equal to 1, which is added to 37653, and there is no remainder.

```
251076872 ÷  6668
125538436    3334
---------   -----
37661|5308 ÷ 10002
    7|5322
-----|----
37653|9986
     |  16
     |-----
     10002
```

NOTE.—It need scarcely be remarked that the remainder in division must be always less than the divisor. In making use of the present methods, however, the remainder frequently comes out equal to the divisor; in such cases 1 is added to the quotient, as in the example, and there is no remainder, the division being exact.

Sometimes the remainder will come out greater than the divisor; in these cases the remainder is divided by the divisor, the result is added to the quotient already found and the difference between said greater remainder and the divisor, will be the final remainder, as shown in example 7, page 23, to which the student is referred. (*See example, page* 250.)

EXAM. 12. Divide 653134680 by 8888.

Here, we add an eighth to each term and obtain 9999 for a simple divisor.

The quotient is 73485, the remainder being equal to 1, which is added to the quotient.

```
653134680 ÷ 8888
 81641835   1111
---------   ----
73477|6515 ÷ 9999
    7|3477
     |   7
-----|----
73484|9999
```

NOTE.—If there be a remainder and it is desirable to find the equivalent decimal, annex as many ciphers as there are decimal places required and continue the division, as in the first part, illustrated in the following:

EXAM. 13. Divide 47032938 by 12501 to five decimal places.

In this, the terms are multiplied by 8 and the divisor is 100008, by which we divide. The quotient is 3762, and the remainder 33408 to which we annex five ciphers for the number of decimal places required, and continuing the division as in the first part, we obtain the decimal .33405, making the quotient 3762.33405, correct to five decimal places.

```
 47032938 ÷  12501
----------   ------
3762|63504 ÷ 100008
    |30096
    |-----
    |33408|00000
    |    2|67264
    |-----|-----
    .33405|32736
```

NOTE 1.—By annexing ciphers to each successive remainder the division can be carried to any desirable length.

2. If there be decimals in the dividend annex them to the remainder instead of ciphers, supplying the deficiency by ciphers, if necessary, to correspond with the number of decimal places required, as illustrated in the following:

EXAM. 14. Divide 183170503.6432 by 24995 to six decimal places.

```
 183170503.6432 ÷ 24995
 7326 82014.5728   99980
    1|46520
      40
 7328|285745|72800
           57|14900
              1140
     .285802|88840
```

Here, the terms are multiplied by 4 and the divisor becomes 99980, a simple one, the complement being 20. Multiplying 7326 by 20, the result is set to the right beginning at the units' place; next, 20 times 2 (1 plus 1 carried in adding) are 40, set in proper position, and by addition, we find 7328 for quotient, and 28574 for remainder, to which 5728 is annexed, also two ciphers to make six places to correspond to the number of decimal places required. Multiplying 285745, now, by 20 and 57, also, by 20 and adding, we obtain .285802, true to six places, leaving 88840 for remainder. The quotient is 7328.285802+. And by annexing ciphers to the remainder, and continuing the division, any required number of decimal places may be obtained.

NOTE.—It is worthy of remark, that any part or any multiple of a simple divisor can also be simplified when used as a divisor. Thus, if $\frac{1}{3}$ of 99980= $33326\frac{2}{3}$ be used as divisor. To divide by $33326\frac{2}{3}$ it is multiplied by 3 to get 99980; and if $\frac{1}{8}$ of 99980=$12497\frac{1}{2}$, or 12497.5 be used, multiplying either number by 8 gives 99980.

Again, if one-half of 12501=$6250\frac{1}{2}$, or 6250.5, be used as divisor; multiplying by 2, and the result by 8, will give 100008; if 4 times 12501=50004 be used: annexing a cipher to the latter we obtain 500040, and dividing this by 5, gives 100008, a simple divisor, and so of other numbers. (*For further illustrations, see page* 250.)

EXAM. 15. Divide $5296765 among 3001 persons.

```
  5296|765 ÷ 3001
 $1765|588+
      |588+
```

Here, we cut off the three right hand figures, 765, the result, 5296|765, is the quotient for 1000. Then, dividing this by 3, we obtain 1765|588+ (*the decimal may be continued if desirable*), the quotient for 3000. To rectify the result which is too large, 3000 being less than the true divisor, we subtract the 1-3000 part of 1765, or .588+, and we obtain $1765, the required quotient.

To get the 1-3000 part of 1765, remove the decimal point three places to the left and divide by 3. (1.765÷3=.588+.)

EXAM. 16. Divide 21187060 by 12004.

In this, we divide the terms by 4, and the divisor becomes 3001, a simple one, by which we divide as in the foregoing example, the quotient is 1765.

$$\begin{array}{r|l}
\multicolumn{2}{l}{21187060 \div 12004} \\
5296 & 765 \div 3001 \\
1765 & 588+ \\
 & 588+
\end{array}$$

From the foregoing examples and illustrations it will be seen that the division can be readily simplified when the divisors run as follows:

1202, 1203, 1204, 1206, 12012, etc.; 12002, 12003, 12004, etc.; 1402, 1407, 14014, etc.; 1602, 1604, 1608, 16016, etc.

Also such as

1212, 1414, 1616, 2424, 3636, 4848, 5656, 6464, etc.

If it were required to divide, for instance, by

2408, 24008, 240008, etc., we would divide by 8 and get 301, 3001, 30001, etc., for new divisors, and the division would be as in the last example, and so with the other numbers.

To divide by 3636, for instance, we would first divide by 6, getting 606; dividing this in turn by 6 we get 101 for new divisor, and so with similar combinations, always bearing in mind that

Whatever operation is performed on the divisor to simplify it, the same operation must be performed on the dividend also.

PROBLEMS SOLVED BY RULE III.

EXAM. 1. If 792 milch-cows be bought for $59400, what is the average price of each?

$$\begin{array}{r|l}
\multicolumn{2}{l}{59400 \div 792} \\
74 & 25 \div 99 \\
 & 74 \\
 & 99
\end{array}$$

To solve this problem, we divide the whole cost by the number of cows, thus:

A slight inspection of the divisor, 792, shows that by dividing it by 8, we obtain a simple divisor, 99, by which we can readily divide according to Rule III. Dividing both dividend and divisor, then, first by 8, we next divide the new dividend, 7425, by the new divisor, 99, getting 74 for quotient and 99 for remainder. The quotient is, then, \$$74\frac{99}{99}$, or rather, \$75, the required price.

Now, as the mental eye cannot always readily tell whether such a number as 792, for instance, is a multiple of some particular number, as 8, the following will be found a much more rapid way of simplifying such numbers when presented as divisors:

Let us take 800, and call it the *approximate* divisor, in connection with 792, the *real* divisor, arranging them as shown in the margin, and using periods, or dots, instead of the ciphers in 800. The difference of the divisors is 8.

$$\begin{array}{r} 8.. \\ 792 \\ \hline 8 \end{array}$$

Examining the process, now, in the foregoing example, it will be seen that we first divided by 8, and then by 100, in other words, we divided by 800, using the component factors, 8 and 100 ($8\times100=800$), getting 74 for quotient, and 25 for remainder. This result is too small since we have divided by 800, instead of 792. To rectify, we have to multiply the quotient, 74, by 8 (the difference of the divisors), and divide the product, 592, by 800. (Gen. Prin. III, page 18.)

Now, multiplying 74 by 8, and dividing by 800, is the same as multiplying by 1, and dividing by 100 ($\frac{8}{800}$ being equal to $\frac{1}{100}$), and that is exactly what was done in the example; we simply added $\frac{1}{100}$ of the quotient, 74, that is, .74, or rather |74, the vertical line being used instead of the point. The price is, then, \$$74\frac{99}{99}$, or rather \$75.

It will be seen upon examination, also, that we have simply divided the eighth part of the total price by the eighth part of the number of cows, that is, we have divided $7425 by 99. To divide by 99, we use 100 for approximate divisor, and 1 (100 — 99) is the complement; so we add $\frac{1}{100}$ of the quotient, .74, or |74.

NOTE.— *Complement*, in the language of Arithmetic, is what any number wants of being a unit of the next higher order. In the above example, 99 wants 1 of being 100, and we call 1 the complement. If 93, 993, or 9993, 7 would, in each case, be the complement, etc.

EXAM. 2. If the taxes paid by 6986 persons amount to $528351.18, what is the average tax to each?

```
               7...
528|351.18 ÷ 6986
 75|478.74     14
   |150
   62|874
     |124
     |998
```

Taking 7000 for approximate divisor, in this, and arranging the divisors, as in example 1, we find their difference to be 14.

To divide by 7000 now, it is immaterial whether we divide first by 7, and then by 1000, or divide by 1000 first and then by 7, the result being the same in either case. We will choose the latter course. Cutting off three figures from the right of the dollars divides by 1000. Then, extending the vertical line, we divide by 7, getting $75 for quotient and $478.74 for remainder, or $75|47874 as quotient for 7000. The result obtained being too small, having made use of too large a divisor, we have to multiply the quotient, 75, by 14 (the difference of the divisors), and divide the product by 7000. Multiplying 75 by 14, and dividing by 7000, is the same as multiplying by 2 and dividing by 1000 ($\frac{14}{7000} = \frac{2}{1000}$); and as

the vertical line performs the division by 1000, all we have to do is, simply to add 2 times 75, or 150, placed in proper position.

There is now a remainder of $628.74, or 62874 cents, in all, to be still divided. Now, as the remainder in Division is always a part of the dividend, it is evident that 62874 is part of the *new* dividend, $75|47874; consequently, in dividing 62874 cents, there is no need of dividing again by 7 (the original dividend having been divided by 7), so we simply cut off three figures, 874, from the cents, and multiply 62 (to the left of the line) by 2, as was done with $75, and setting the product, 124, to the right, we add. The quotient is now $75.62, and a remainder of 998. Since the original dividend has been divided by 7, it is evident that 998 is only the seventh part of the *true* remainder. Multiplying 998 by 7 gives the true remainder, 6986, which contains the true divisor once; 1 is added to 62 cents, making $75.63, the required average tax.

REMARKS.

Remarks on Rule III. — If the student have followed the thread of our reasoning thus far, he will readily see, on analyzing the foregoing example, that we have simply divided the seventh part of the dividend by the seventh part of the divisor; that is, we have divided $75478.74 by 998 (6986 ÷ 7). To divide by 998 we use 1000 for approximate divisor, and add the complement, or 2 times the quotient; 2 times $75 in the first case, and 2 times 62 cents in the other. This gives $75.62 and a remainder of 998 for quotient; fully expressed, $75.62\frac{998}{998}$, or rather $75.63.

From a due consideration of the foregoing results we see that: *When the divisor is such that the difference between it and the next higher number ending in ciphers is divisible by the significant part of such higher number, the complement, or multiplying figure, will be the result obtained by dividing said difference by the said significant part.*

Referring to problem 1, for instance, where we have taken 800 for approximate divisor, in connection with 792, the true divisor, we see that the difference is 8, and that said difference contains 8,

the significant figure of 800, once; 1 is the complement or multiplier.

Again, referring to problem 2, we see that 7, the significant figure of 7000, is contained 2 times in 14, the difference of the divisors; 2 is then the complement or multiplier.

For brevity we will henceforth call the multiplier in such cases the *key*.

The student's attention is now invited to the following: Taking 800, as was taken in the remarks on Rule II, we see that every number between that and 700, differing by 8, can be readily simplified when used as a divisor.

Thus, 800, 792, 784, 776, 768, 760, 752, 744, 736, etc.

Again, if we take the numbers between 8000 and 7000, 80000 and 70000, 800000 and 700000, etc., a similar relation will be found to exist.

Thus, 8000, 7992, 7984, 7976, 7968, 7960, etc.; 80000, 79992, 79984, 79976, 79968, etc.; 800000, 799992, 799984, 799976, 799968, etc.

Suppose it were required to divide by 7968, for instance, we would take 8000 for approximate divisor, and both divisors would stand thus:

$$\begin{array}{r} 8\ldots \\ 7968 \\ \hline 32 \end{array}$$

We would now divide by 8000, as in problem 2, and 4 (32 ÷ 8) is the key.

And if the divisor were 79968, we would use

$$\begin{array}{r} 8\ldots. \\ 79968 \\ \hline 32 \end{array}$$

where 80000 is the approximate divisor, and 4 also the key; and so of the other numbers.

And if the numbers between 900 and 800, 700 and 600, 600 and 500, 500 and 400, 400 and 300, etc.; also between 9000 and 8000,

90000 and 80000, etc.; 8000 and 7000, 80000 and 70000, etc., be compared, a similar relation will be found to exist.

If it were required to divide by 2982, for instance, the divisors would stand thus:

$$\begin{array}{r} 3\ldots \\ 2982 \\ \hline 18 \end{array}$$

3000 is approximate and 6 the key.

Reviewing the numbers, now, which can be readily simplified, when presented as divisors, let us take any series, say from 700 to 800, placing those to which Rule II is applicable in a column to the left, and those to which Rule III is applicable in a column to the right, as below, and the key for each in a column between, thus:

Rule II.	Key	Rule III.
700		800
707	1	792
714	2	784
721	3	776
728	4	768
735	5	760
742	6	752
749	7	744
756	8	736
763	9	728
770	10	720
777	11	712
784	12	704
791	13	696
798	14	etc.

To divide by 756, for instance, we would apply Rule II; first dividing both dividend and divisor by 7, we would then divide

the seventh part of the dividend by the seventh part of the divisor, which, in this case, would be 108. The approximate divisor for 108 would be 100, and 8 the key.

And to divide by 736, for instance, it would stand thus,

$$\begin{array}{r} 8.. \\ 736 \\ \hline 64 \end{array}$$

800 would be the approximate divisor, and 8 the key (64÷8), and so of the other numbers.

When the *difference* between the real and the approximate divisor is *less* than the significant part of the latter, we indicate the division by the fractional form; illustrated in the following:

EXAM. 3. It required \$563650.88 to pay a certain army, giving each man \$23.92; how many men were in that army?

$$\begin{array}{r|l|l}
 & & \quad 24.. \\
563650 & 88 & \div\ 2392 \\ \hline
140912 & 72 & \quad \tfrac{8}{24} = \tfrac{1}{3} \\ \hline
23485 & 4533+ & \\
78 & 2833+ & \\
 & 26 & \\ \hline
23563 & 99 & 66 \\
 & & 33
\end{array}$$

To solve this, we have to find how many times \$23.92 is contained in \$563650.88.

NOTE.— The arithmetical student need scarcely be told, that operations on decimals are performed as on whole numbers, due attention being paid to the proper position of the decimal point.

Moving the decimal point in each two places to the right, in other words, calling both cents, throws off the decimals, and we have whole numbers at once.

To divide by 2392, we take 2400 for approximate divisor; the difference is 8, which does not contain 24, the significant part of 2400, so we indicate such division by the fraction $\frac{8}{24}$, which is equal to $\frac{1}{3}$, and this is the key or multiplier.

We now divide by 2400, first by 100 by means of the line, and next by 24. To divide by 24 we use the component factors 4 and 6 ($4 \times 6 = 24$), dividing first by 4 and then by 6, and we get 23485 for quotient and .4533+ for remainder, 3 being repeated without end.

We have now to add 8–2400, or $\frac{1}{300}$ part of the quotient, 23485, or, what amounts to the same thing, the $\frac{1}{100}$ part divided by 3. The hundredth part of 23485 is 234.85, and the third part of the latter is 78.2833+, 3 being repeated without end. Next, the $\frac{1}{300}$ part of 78, or .26, is added.

Now, since the vertical line performs the division by 100, right through, all we have to do is simply divide the partial quotients, 23485 and 78, each by 3, setting the results in proper position.

Adding the several results now, we get 23563 for quotient and .9966+ for remainder, 6 being repeated without end.

To find the correct decimal, we continue the process, simply cutting off 66 and adding a third of .99, or .33, placed in proper position (that is the $\frac{1}{300}$ part of .99, or .0033, this being the value of |33 as it now stands in the work). The remainder is now .9999+, or .9 repeated to infinity, which is equal to 1. Adding 1 to 23563 gives 23564, the number of men, and there is no remainder.

NOTE.—To show that .9999, etc., is equal to 1: If we take the digit 9 as divisor, and any one of the remaining eight digits as dividend, and express the value of such division in the language of decimals, we will, in every case, obtain a repetition, without end, of the figure of the dividend. Thus, $\frac{1}{9} = .11111$, etc.; $\frac{7}{9} = .7777$, etc.; and

Conversely, .11111, etc., $=\frac{1}{9}$; .7777, etc., $=\frac{7}{9}$; and on the same principle, .9999, etc., is equal to 1; that is, .9999, etc., $=\frac{9}{9}$, or 1, as in the foregoing example. Hence,

To find the value of a decimal repeated to infinity: Set down the repeated figure or figures for numerator, and 9, or as many nines, for denominator, as there are figures repeated, and the result is a common fraction equal in value to the decimal. Thus, .3333, etc., $=\frac{3}{9}$, or $\frac{1}{3}$; and .9696, etc., $=\frac{96}{99}$, etc.

EXAM. 4. Suppose \$5083763.16 were to be divided among a number of persons, giving each \$398.32; how many persons would receive that sum?

$$\begin{array}{rr|rcl}
 & & & & 4\ldots\ldots \\
 & 50837 & 6316 & \div & 39832 \\
 & \overline{12709} & \overline{4079} & & \overline{\tfrac{168}{4}} = 42 \\
76254 & & & & \\
318 & 53 & 3778 & & \\
 & & 2226 & & \\
 & & 42 & & \\
 & \overline{12763} & \overline{0125} & &
\end{array}$$

Here we take 40000 for approximate divisor. The difference of the divisors, 168, divided by 4, the significant part of 40000, gives 42, the key. Cutting off four figures, 6316 (one figure for each cipher in 40000), from the right of the dividend, and then dividing by 4, gives the quotient for 40000, or 12709|4079.

We have now to add 42 times 12709. To multiply by 42, we use the component factors 6 and 7 ($6\times7=42$). Setting 6 times 12709, or 76254, a little to the left, as shown in the margin, we multiply the latter in turn by 7, setting the product, 533778, in proper position, as the rule directs. Next, we add 42 times 53, or 2226, placed in proper position; thus: 6 times 53, or 318, is set to the left, as in the margin; then 7 times 318, or 2226.

A short inspection, now, shows that in adding the remainders, or numbers to the right of the line, 1 is to be carried to the left, or quotient's place; this 1 is also multiplied by 42, and the pro-

duct set in proper position. Addition now gives 12763 persons, and .0125 remaining, which is equal to $5.

NOTE.—To find the value of the remainder, .0125, in such examples as the foregoing, it must be carefully borne in mind, that the remainder, in Division, is always a part of the dividend; and that whatever operations are performed on the dividend and divisor, by way of preparation to simplify the division, the *reverse* of these operations is performed on the remainder, to find the true remainder.

In the foregoing example, the dividend has been divided by 40000, consequently, .0125 is only the 40000 part of what it ought to be. To restore it to its proper value; that is, to the same denomination as the dividend, we multiply it by 40000. To multiply .0125 by 40000, we simply move the decimal point four places to the right, which gives 125, the product for 10000; multiplying 125, then, by 4 gives 500, or the product for 40000, the true remainder.

Now, since the dividend in the example is cents, the true remainder, 500, is also cents. The value of .0125 is, therefore, $5.

When the divisor can be conveniently resolved into component factors, we proceed by successive division; illustrated in the following:

EXAM. 1. Divide 661740804 by 17946.

Taking 18000 for approximate divisor, we see that the difference between that and the real divisor is 54, which contains 18, the significant part of the approximate divisor, without a remainder; and this being the case, the divisor, 17946, will also contain 18 without a remainder.

$$\begin{array}{r} 18000 \\ 17946 \\ \hline 54 \end{array}$$

Now, the factors of 18 are 3 and 6 ($3\times6=18$). Dividing 17946 by 3, and the result by 6, we obtain 997, a simple divisor.

$$\begin{array}{r|r} 3 & 17946 \\ 6 & 5982 \\ \hline & 997 \end{array}$$

The factors of 17946, now, are 3, 6 and 997, by which we divide in succession as shown in the margin. The quotient is 36873, and the remainder, 997, which contains 997 once more, 1 is added to the quotient, making 36874, and there is no remainder. (*See exam.* 8, *page* 23.)

$$\begin{array}{r|r|l} 3 & 661740804 & \\ 6 & 220580268 & \\ \hline & 36763 & 378 \div 997 \\ & 110 & 289 \\ & & 330 \\ \hline & 36873 & 997 \end{array}$$

NOTE.—If 18 be taken from 18000, then from the remainder and from each successive remainder, the numbers thus found, viz., 17982, 17964, 17946, 17928, 17910, 17892, etc., each differing by 18, can be treated similar to the example shown, if presented as divisors. (*For further illustrations, see examples from page* 249 *to* 253.)

EXAM. 2. Divide 35372903712 by 240168.

In this, a slight inspection shows that 168, the right hand figures of the divisor, is a *multiple* of 24, the left hand figures; and the factors of 24 are 4 and 6. Dividing 240168 by 4, and the result by 6, we obtain 10007, a simple divisor. The factors, now, of 240168 are 4, 6 and 10007 by which we divide as shown in the margin. The quotient is 147284. the remainder 10007 being equal to 1 ($\frac{10007}{10007}$). (*See Rule II, page* 15, *and exam.* 2. *p.* 13.)

```
4|240168
6| 60042
 | 10007

4|35372903712
6| 8843225928
  147387| 0988 ÷ 10007
     103| 1709
  147283| 9279
        |  728
        |10007
```

NOTE.—It will now be readily seen that, when the divisor is not too large, and when the last figures are a multiple of the first figures, said first figures being easily factored, the foregoing method can be applied; such numbers as: 24168, 32192, 320192, 3200192, 48240, 480240, 72648, 720648, 4590, 45090, 450135, 450180, 450225, etc., etc.

GENERAL SHORT METHOD FOR ALL NUMBERS.

When the foregoing methods cannot be conveniently applied, the following short method, cutting off 50% of the usual work, can be used, *and may, perhaps, be preferred in all cases;* illustrated in the following:

EXAM. 1. Divide 17653762 by 3658.

Here, we take 17653 for the first partial dividend, and drawing a line to the right of 3, we find 4 for the first figure of the quotient. We now multiply 3658 by 4; but instead of writing down the product and subtracting, we simply add enough to each product, as we proceed, to give the figure of the partial dividend.

```
3658)17653|762(4826.
      3021|7
        95|36
        22|202
          |254
```

Thus, 4 times 8 are 32 and 1 (setting down 1) are 33 which gives 3 of the dividend; 3 to carry; 4 times 5 are 20 and 3 are 23; and 2 (required to make 25) is set down under 5 of the dividend: 4 times 6 are 24, and 2 (carried from 25) are 26, set down 0, the figure of the dividend being 6; carry 2; 4 times 3 are 12, and 2 are 14, and 3 is set down to make 17. The number 3021 is the remainder, to which the next figure, 7, is brought down; giving 30217 for next dividend.

(*Continued on page* 46.)

```
3658)17653|762(4826
      3021|7
        95|36
        22|202
     -----|----
          |254
```

Now, 3658 is contained 8 times in 30217: Say 8 times 8 are 64, and 3 (set down to make 67): carry 6; 8 times 5 are 40, and 6 are 46, and 5 (set down to make 51): carry 5; 8 times 6 are 48, and 5 are 53, and 9 (set down to make 62): carry 6; 8 times 3 are 24, and 6 are 30, nothing set down, the number above being 30; the remainder is 953 to which 6, the next figure of the dividend is brought down, giving 9536 for partial dividend. The next figure of the quotient is 2: say 2 times 8 are 16, set down 0, the figure immediately above being already 6: carry 1; 2 times 5 are 10, and 1 are 11, and 2 (set down to make 13): carry 1; 2 times 6 are 12, and 1 are 13, and 2 (set down to make 15): carry 1; 2 times 3 are 6, and 1 are 7, and 2 (set down to make 9): Proceeding thus, we find the next figure of the quotient, 6, and the remainder is 254.

NOTE.—If it should be necessary to continue the work into decimals, annex ciphers as in the usual method, and proceed as before.

When the divisor ends in ciphers we would proceed as in the following:

EXAM. 1. Divide 6569437124 by 998000.

```
6569|437|124 ÷ 998|000
  13|138
    | 26
----|------
6582|601124
```

Here, we cut off the ciphers from the divisor, and as many figures (124) from the right of the dividend for the last figures of the remainder. Then, dividing the remaining part of the dividend by the remaining part (998) of the divisor, we get 6582 for quotient and .601 for remainder. The figures cut from the divi-

dend are now brought down, and the true remainder is 601124. The manner of finding the correct decimals of the remainder has been already pointed out. The decimal here found is correct to two places (.60).

When the digits of the divisor are all the same figure we would proceed as in the following:

EXAM. 1. Divide 3009581015 by 77777.

30095	81015	÷ 77777
4299	40145	11111
38694	61305	÷ 99999
	38694	
	99999	

NOTE. — It will be readily seen that, when the digits of the divisor are all the same figure, a succession of 1's is obtained by dividing the said divisor by one of its digits; and multiplying this quotient of 1's in turn by 9, gives a succession of 9's by which we can readily divide according to Rule III.

Dividing 77777, in this example, by 7 gives 11111, and multiplying this by 9 gives 99999, a simple divisor. Performing the same operations on the dividend gives 3869461305 for a new dividend. Dividing this by 99999 gives 38694 for quotient and 99999 for remainder, which, being equal to the divisor, adds 1 to the quotient, making 38695, and there is no remainder.

NOTE. — By referring to the process in the foregoing example, we see that the number of figures to be cut from the right of the dividend in such cases is equal to the number of digits in the divisor, thus giving the position of the vertical line before commencing operations on the dividend.

HINTS FOR THE STUDENT.

From the foregoing example it will be seen that, *when the divisor is such that, being divided by a measure, or*

exact divisor, we obtain a repetition of any of the nine digits, the division can be at once simplified.

This will be made clear by the following observations: Let us take the digits 2, 3, 4, 5, etc., repeated two times, three times, etc., or any multiple of such repeated digits, and examine them carefully for a few moments, thus:

$22 \times 6 = 132$	$222 \times 6 = 1332$	$2222 \times 6 = 13332$
$33 \times 5 = 165$	$333 \times 5 = 1665$	$3333 \times 5 = 16665$
$44 \times 4 = 176$	$444 \times 4 = 1776$	$4444 \times 4 = 17776$
$55 \times 7 = 385$	$555 \times 7 = 3885$	$5555 \times 7 = 38885$
$66 \times 3 = 198$	$666 \times 3 = 1998$	$6666 \times 3 = 19998$
$77 \times 2 = 154$	$777 \times 2 = 1554$	$7777 \times 2 = 15554$
$88 \times 3 = 264$	$888 \times 3 = 2664$	$8888 \times 3 = 26664$
$99 \times 3 = 297$	$999 \times 3 = 2997$	$9999 \times 3 = 29997$

Suppose, now, it were required to divide by any of the above numbers, say 132, 1332 or 13332; we see by inspection that if these be divided by 2, 3, 4, 6 or 12, we obtain a repetition, in every case, of a particular digit: 66, 666, 6666; 44, 444, 4444; 33, 333, 3333; 22, 222, 2222; 11, 111, 1111; any one of which can be treated the same as 77777 in the last example.

And the same is true of the remaining numbers, 165, 176, 385, etc.

Again, if any multiple of those multiples be used as a divisor, such as

$132 \times 3 = 396$	$1332 \times 3 = 3996$
$165 \times 3 = 495$	$1665 \times 3 = 4995$
$297 \times 3 = 891$	$2997 \times 3 = 8991$

etc., the same treatment is applicable; but for most of such multiples more rapid methods have been already pointed out.

To divide by 396, 495, 2997 or 8991, for instance, we would make use of the next higher numbers ending in ciphers, as 400, 500, etc., for approximate divisors, getting the key at once, as has been already explained. (See remarks on Rule III.)

NOTE. — Dividing the divisor by one of its digits, when the figures are the same, and then multiplying by 9, we need scarcely remark, is the same as multiplying first by 9, and dividing by the digit after. To divide by 1332, for instance, we first divide by one of its measures, or an exact divisor, say 2, getting 666, and this in turn by 6, getting 111, which is multiplied by 9, getting 999 for a simple divisor. But dividing by 2 and then by 6 is dividing by 12 ($2 \times 6 = 12$). Now, by first multiplying 1332 by 9, and dividing after, the process will be more readily performed; thus, $1332 \times 9 = 11988$.

Taking 12000 for approximate divisor now, in connection with 11988, and arranging them as has been already pointed out, they will stand as in the margin, the key being 1, and the division being the same as above; that is, by 2, 6 and 1000; or, by 1000 first, and then by 12, as in the last arrangement. In such cases, then, it is perhaps preferable to multiply first by 9.

```
   12...
   11988
   -----
      12
```

When the digits of the divisor are all the same figure and a cipher intervenes, as 10101, 20202, 3030303, etc., we would proceed as in the following:

EXAM. 1. Divide 3045261828 by 80808.

```
 3045261828.. ÷ 80808..
 ------------   -------
 301480|920972 ÷ 7999992
 ------|------
  37685|115121+
       | 37685
       |------
       |152806
```

Here, we see at a glance, that, if the divisor, 80808, be multiplied by 11, we get 888888, by which we can readily divide as in the previous example, first dividing by 8, and then multiplying by 9.

Now, multiplying first by 11, and afterward by 9, is multiply-

ing by 99. So we prefer to multiply 80808 by 99 first, getting 7999992 for a new divisor.

To multiply by 99, we conceive two ciphers, represented by dots, or periods, annexed to the divisor, as seen in the margin; this multiplies it by 100, giving 80808.. as the result. From this we subtract 80808, without setting down the latter, however, but simply setting down the difference, 7999992. thus: 8 from 10 (represented by the first dot to the right) and 2; carry 1 to 0 (to the left of right hand 8), 1 from 10 (the second dot), and 9; carry 1 to 8 (the second or middle 8), 9 from 18, and 9; and so on, all from the expression 80808..

Going through a similar process with the dividend, that is, subtracting 30452618 from 30452618.., without setting down the first (being already contained in the last expression), we get a new dividend which is divided by 7999992; 8000000 being the approximate, and 1 the key, giving 37685.152806 for quotient, true to six places of decimals (bearing in mind that the line is used for the decimal point).

NOTE.— When two or more ciphers intervene, as 1001001, 7007007, etc., we multiply by 999, 9999, etc. (short method), always one 9 more than the number of intervening ciphers, to simplify the division. (For short method of multiplying by any number of 9's, see Contractions in Multiplication, page 74.)

When the figures of the divisor are repeated; thus, 212121, 323232, etc., we would proceed as in the last case, as illustrated in the following:

EXAM. 2. Divide 16299801882 by 212121.

```
   16299801882.. ÷ 212121..
 3)1613680|386318 ÷ 20999979
  7)537893|462106
     76841|923158
          | 76841
          |999999
```

A moment's glance at the divisor, in this example, shows that, if it be divided by 3 or 7, we get 70707, or 30303, and the division can be performed as in the previous example.

Multiplying both dividend and divisor, then, by 99, the new divisor becomes 20999979, with which we use 21000000 as approximate divisor, the difference being 21 and the key 1.

The quotient is 76842, there being no remainder.

NOTE. — There are two ways for finding the value of the remainder .999999.

First. Reverse the process performed on the dividend; that is, divide by 99 and multiply by 21 (3×7). Dividing 999999 by 99, we get 10101, and multiplying this by 21 gives 212121, which contains the divisor once; .999999, therefore, is 1.

Second. Annex ciphers and continue the division, as has been already pointed out, and 9 will be repeated to infinity; but it has been already shown that .9 repeated to infinity is equal to 1.

Whenever, therefore, .9, .99, etc., is remainder in such cases, add 1 to the quotient and there is no remainder.

OTHER SIMPLE METHODS.

It must be carefully borne in mind that the two principal rules laid down in the beginning of this work are the entire secret to our methods, a little practice enabling us to bring any ordinary number used as a divisor within one or other of those rules. Hence, *the student need not expect to understand what follows unless he has thoroughly posted himself on the groundwork.*

To still further assist the student to master Simplified Division, we subjoin a few more examples illustrating our methods, which, if rightly understood, will prove extremely simple and interesting.

EXAM. 1. If 73 musical instruments cost $12802.74, what did one instrument cost at that rate?

From several methods of simplifying the division by 73, we select the following, as, perhaps, the most simple and expeditious that can be given.

$$\begin{array}{r l}
1280274 & \div\ 73..\\
426758 & \quad 2433+\\
42675.8 & \quad\ \ 243+\\
4267.58 & \quad\ \ \ \ 24+\\
\hline
175\,|\,3975.38 & \div\ 10001\\
175 & \\
\hline
38\,|\,0038 & \\
38 & \\
\end{array}$$

Annexing two ciphers (represented by dots) to 73, in other words, multiplying it by 100, we get 7300. To multiply the dividend by 100, we simply call the dollars cents, and we have 1280274 cents to be divided by 7300. To divide by the latter, we set under it, one-third of itself, one-tenth of that third, and one-tenth of that tenth, and adding, we get 10001 for a simple divisor. Going through a similar operation with the dividend. we get 1753975.38 for new dividend, which being divided by 10001, gives 175 for quotient, and .3800 for remainder, to which decimal 38 is annexed, and continuing the process, we find 38 is the correct decimal, or cents, in this case. The price, then, is $175.38.

NOTE.—If the division of 7300 be continued, we find the decimal to be .333, etc., as shown in the margin, 3 being repeated without end. And in adding a tenth, etc., 3 will be repeated in like manner, so that when the results are added, we get 10000.999, etc. But it has been shown (note, page 42) that .999, etc., is equal to 1; therefore, 10000.999, etc., = 10001. Hence,

$$\begin{array}{r}
7300\\
2433.333+\\
243.333+\\
24.333+\\
\hline
10000.999
\end{array}$$

To divide by 73 *then: Multiply the dividend by* 100 (*simply conceive two ciphers annexed*), *and under the product write one-third of itself, one-tenth of that third, and one-tenth of that tenth, and divide the sum by* 10001.

EXAM. 2. The annual income of a certain American citizen is $1538475; what is his income for one day?

```
1538475   ÷ 365
3076950.  ÷ 730.
10256500
 1025650
  102565
4215|4215
    |4215
```

To divide by 365 we double it, then double the dividend, and we have 3076950 to be divided by 730. Annexing a cipher (dot) now, to both dividend and divisor, we get 7300 for new divisor, by which we divide as in the previous example, and we get $4215, the income for one day.

EXAM. 3. If the aggregate annual tax paid by 73000 tax payers be $31220640, what is the average individual tax?

```
312|20640  ÷  73000
104|06880     24333
 10|40688      2433
  1|04068       243
427|72276  ÷ 100010
   |  4270
   |68
```

Here, we set under both dividend and divisor one-third of each, one-tenth of that third, and one-tenth of that tenth, and adding, we have 42772276+, to be divided by 100010. The quotient is found by Rule II to be $427.68, the average tax.

NOTE.—We see by inspection that 68 is the correct decimal, or number of cents; for if we complete the subtraction and continue the process we

get .68006, to which we annex 80, the figures omitted in the division by 10, and we have then .6800680 to be divided by 100010, and the result is .68, as found in the example.

To divide by 73000, then, we have the following simple rule: *Under the dividend write one-third of itself, one-tenth of that third, and one-tenth of that tenth, add the four lines together and divide the sum by* 100010.

NOTE. — By continuing the decimals, in the division of 73000 by 3 and 10, it will be seen that 3 is repeated to infinity, and the sum of the four lines is 100009.9999, etc.; but this has been shown to be equal to 100010. (See note to example 3, page 42.) It is optional whether the vertical line be drawn before taking the parts or when commencing to divide by 100010, as, knowing the divisor, we know the number of figures to be cut from the right of the dividend, namely: five.

This simple and expeditious method of dividing by 73000 will be of great value in solving problems in interest, for days, at any rate per cent, on the basis of 365 days to the year. (Explained in the article on Interest.)

EXAM. 4. If 732 building lots be valued at \$356484, what is the average price of each lot?

```
 356484.  ÷  732.
 1188280     2440
  118828      244
 ───────    ─────
 487|1948 ÷ 10004
    |1948
```

Here, we simply conceive a cipher annexed to both dividend and divisor, and under each result set one-third of itself, then one-tenth of that third, and adding we have 4871948 to be divided by 10004. The quotient by Rule II is \$487, the average price of each lot.

A knowledge of the methods here given, will be of particular advantage in operations where the divisor is not subject to change, as, for example 43560 sq. ft. to an acre; 5280 feet to a mile; 625 sq. l. in a pole; 7.92 in. to a link; $24\frac{3}{4}$, or 24.75 cub. ft. to a perch of masonry; 2240 lbs. to a gross ton; 144 articles to a gross, &c.

In problems on Percentage the methods will be found extremely valuable where the divisors run like 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, &c.

EXAM. In 76824763 sq. ft. how many acres, true to five places of decimals?

NOTE.—Here, we have to divide by 43560, and a slight inspection shows that the component factors are 40, 11 and 99 ($40 \times 11 \times 99 = 43560$.)

4.0	4356.0
11	1089
	99

Pointing off one figure and dividing by 4 divides by 40; next, we divide by **11**, getting 174601.734. This is now divided by 99 by simply pointing off two places for 100, and adding for the complement 1, once 1746 and once 17 set in proper position, the result is 1763|64734

7682476.3	
1920619.075	
1746	01.734
17	46
	17
1763	647 34
6	47
	6
	.65387

To find the complete decimal, now, we divide 64734 by 99; simply adding once 647 and once 6; the answer is 1763.65387 acres true to five places of decimals.

NOTE.—To divide by 5280; the component factors are 60, 8 and 11; make use of successive division, decimally. To divide by 625, multiply by 4 and the result by 4, and divide by 10.000 ($625 \times 4 = 2500$, and $2500 \times 4 = 10000$.) bearing in mind to multiply the dividend also, to preserve the relation. For 792 take $\frac{1}{8}$, of the dividend and divide by 99 ($792 \div 8 = 99$.) To divide by $24\frac{3}{4}$, or 24.75, multiply by 4 and divide by 99 ($24\frac{3}{4}$, or $24.75 \times 4 = 99$.) The component factors of 2240, are 40, 7 and 8 ($40 \times 7 \times 8 = 2240$.) and for 144 multiply by 7 and divide by 1008 ($144 \times 7 = 1008$.) *(See rule and exam. page 225.)*

In many cases where, at first glance, it may appear difficult to simplify the division, a short inspection of the divisor will suggest a method, Take, for instance, the following

EXAM. Divide 49862538 by 7854, true to seven places of decimals.

```
49862538      ÷      7854
 4532958              714
 9065916             1428
---------           -----
6346|1412     ÷      9996
   2|5384
    |   8
----  ----
6348|6804000|0000
    |   2721|6000
    |      1|0884
    |       |   4
     -------------
     .6806722|6888
```

Here, a short inspection of the divisor shows that it is divisible by 11, giving 714; doubling this gives 1428; the sum of the three numbers gives 9996 for a simple divisor. Dividing the dividend by 11, in like manner, doubling the result and adding, we have 63461412 to be divided by 9996. Dividing now by 10000, and adding for the complement 4 times 6346 and 4 times 2 we get 6348|6804.

To get seven places of decimals, now, we annex seven ciphers (one for each decimal required) and dividing 68040000000 by 9996, as in the first part of the example, we get 6348.6806722, as required.

NOTE.—It need scarcely be observed that the division may be carried to any length by simply annexing as many ciphers as there are decimals required, dividing in each case by 9996, the new divisor.

Suppose it were required to find five more decimal places; annex five ciphers and divide 688800000 by 9996.

(*See examples on pp.* 58 *and* 59.)

The following simple rule will be found useful when the cost per gross is given to find the cost of a single article:

RULE.—*Multiply the cost per gross (up to $142) by 7; point off three figures for decimals, and the result is the cost of a single article, near enough for practical purposes.*

EXAM. If a gross of padlocks cost $23.50; what is the cost of 1? Answer 16c.

Thus: $23.50 \times 7 = 164.50$ and pointing off three places we have .16450, or 16 c. for business.

EXAM. What is the cost of 1 article at $19 per gross? Answer 13c.

Thus: $19 \times 7 = 133$, and pointing off three places we have .133, or 13c. for business.

Reason.—To divide by 144 we multiply by 7 to get 1008 for a simple divisor; and multiplying 19, also by 7, to equalize, we have $133 \div 1008$; and we simply divide by 1000 instead of 1008 which gives the result near enough when the cost does not exceed $142 per gross.

NOTE.—If greater accuracy be required set 8 times 133 three places to the right and deduct, thus:

	133...
	1064
and the result is correct to four places, viz. .1319	.131936

And if it should be required to find the result correct to six places, we annex six ciphers, or one cipher for each decimal required, and divide by 1008.

EXAM. Divide $19 by 144, true to 6 places of decimals.

Multiplying both terms by 7 we have $133 \div 1008$ true to 6 places of decimals, thus:

Annexing 6 ciphers we apply	133000 | 000 ÷ 1008
	1064 | 000
rule 2 (page 15) and we get	131936 | 000
	8 | 512
.131944, true to 6 places	.131944 | 512

DECIMALS.

It has been already remarked that operations on decimals are performed as on whole numbers, due attention being given to the point, or characteristic of the decimal.

To illustrate, we will take a few of the most important numbers used in Practical Mathematics.

EXAM. 1. Divide 4985.7192 by .7854.

$$
\begin{array}{rl|l}
\multicolumn{2}{r}{\underline{49857192}} & \div \ \underline{7854} \\
\multicolumn{2}{r}{\underline{7122456}} & \quad \underline{1122} \\
6474 & 96 & \div \ 102 \\
\underline{129} & \underline{48} & \\
\underline{6345} & 48 & \\
\underline{2} & \underline{58} & \\
6348 & 06 & \\
 & \underline{06} &
\end{array}
$$

Moving the decimal point in both dividend and divisor four places to the right; in other words, multiplying each by 10000, we have whole numbers. A moment's inspection of the divisor, now, shows that it is divisible by 7, giving 1122, which, in turn, is divisible by 11, giving 102 for a simple divisor.

Dividing the dividend, now, by 7, and the result by 11, we have 647496 to be divided by 102. Here Rule II is applicable, and we get 6348 for quotient.

Or thus:

$$
\begin{array}{r@{}l c r}
\multicolumn{2}{r}{49857192} & & 7854 \\
\multicolumn{2}{r}{7122456} & & 1122 \\
\multicolumn{2}{r}{6474960} & & 1020 \\
\hline
6345 & |4608 & \div & 9996 \\
2 & |5380 & & \\
 & |8 & & \\
\hline
6347 & |9996 & &
\end{array}
$$

Under the divisor set one-seventh of itself, or 1122; then conceiving a cipher annexed to this, we have 11220; dividing this by 11, we get 1020, which is set under 1122, and adding the three numbers together, we get 9996 for a simple divisor. Going through a similar process with the dividend, we get 63454608 to be divided by 9996. Here, Rule III is applicable, and we get 6348 for quotient, the remainder, 9996, being equal to 1. Hence, the

RULE. *To divide by* .7854 *: Below the dividend set one-seventh of itself, and one-eleventh of that seventh, setting the latter one place farther to the left than its proper position; add the three numbers together, and divide the sum by* 9996.

EXAM. 2. Divide 4985.7192 by 3.1416.

$$
\begin{array}{r@{}l c r}
\multicolumn{2}{r}{49857192} & \div & 31416 \\
\hline
\multicolumn{2}{r}{12464298} & \div & 7854 \\
\multicolumn{2}{r}{1780614} & & \\
\multicolumn{2}{r}{1618740} & & \\
\hline
1586 & |3652 & \div & 9996 \\
 & |6344 & & \\
 & |9996 & &
\end{array}
$$

Dividing both dividend and divisor in this by 4, we get .7854 for a new divisor, by which we divide according to the foregoing rule, and we get 1587 for quotient.

EXAM. 3. Divide 496.258 by .07958, true to four places of decimals.

Here, we move the decimal point in the divisor five places to the right, and we have 07958., or rather 7958, for a new divisor. Then, as there are only three places of decimals in the dividend, we annex two ciphers to fill the deficiency, and, moving the point five places to the right, we have 49625800 for a new dividend.

```
 49625800 ÷ 7958
 6203|225 ÷ 994¾
   32|565.75
     |168
 6235|9587|5..
        50|331.75
6235.9638|
```

NOTE.—If 8000 be taken as approximate for 7958, and arranged thus: 5¼ (42 ÷ 8) is the key. From this we conclude that if 7958 be divided by 8, a divisor will be obtained lacking only 5¼ of being some power of 10 (in this case 1000).

```
   8...
   7958
   ----
     42
```

Dividing both dividend and divisor, then, by 8, the new divisor becomes 994¾, the complement being 5¼, as shown in note. Here, Rule III is applicable, 5¼ times 6203, or 32|565.75; then 5¼ times 32, or |168, being set in proper position and added, giving 6235 for quotient and |958.75 for remainder.

To find the decimal, now, true to four places, the vertical line is drawn to the *right* of the *fourth* figure, 7, of the remainder, and two ciphers (dots) annexed to the remaining part (so as to give three places, |5.., to the right of the line, as in the first part of the example); then, setting 5¼ times 9587, or 50|331.75, in position, as in the first part, and adding, we get 6235.9638, for quotient, true to four places of decimals (1 being allowed for the part of the decimal cut away).

EXAM. 4. A perch of masonry contains 24¾ or 24.75 cubic feet. How many perches of masonry in 86006.25 cubic feet?

```
86006.25 ÷ 24.75
3440|25|00 ÷ 99|00
  34|40
    |34
3474|99
```

Multiplying both dividend and divisor by 4, in this, we have 344025 to be divided by 99. Here, Rule III is applicable, 100 being the approximate divisor and 1 the key. Dividing by 100, and adding once each partial quotient; that is, the 1–100 part, we get $3474\frac{99}{99}$, or rather 3475, the required number of perches.

NOTE. —If we used the fractional form, $24\frac{3}{4}$, we would multiply by 4 also, 4 times $24\frac{3}{4}$ being 99. Hence, the

RULE. *To divide by* $24\frac{3}{4}$, *or* 24.75 *: Multiply the dividend by* 4, *and divide the product by* 99.

EXAM. 5. Divide 478932673 by 16667 to five places of decimals.

```
    478932673 ÷  16667
28735|96038   ÷ 100002
     |57470
     |38568|.....
     |     |77136
28735.38567|22864
```

Here, we simply multiply both dividend and divisor by 6 (the divisor being nearly one-sixth of 100000), and the new divisor is 100002, by which we divide, getting 28735 for quotient and 38568 for remainder. To find five places of decimals, now, a vertical line is drawn to the right of the fifth figure of the remainder, and as many ciphers conceived to be annexed as there are figures cut from the right of the new dividend, namely, five; then, continuing the multiplication by 2, as before, we get .38567, the number of decimals required. The quotient, then, is 28735.38567.

Upon examination, however, it will be found that the decimal in the present example is not only correct to five places, but it is correct to nine places, viz.: .385672286, and the correction may be made to any required number of decimal places by continuing the process as above. Hence,

To find any particular number of decimal places we have the following simple

RULE. *To the right of as many figures of the remainder as there are decimals required draw a vertical line, and to the figures on the right of said line, if any, annex, or conceive to be annexed, as many ciphers as will make the number of places* (both figures and ciphers) *equal to the number of figures cut from the right of the dividend at first, and continue the process as in the first part.*

NOTE.—Should the remainder not contain a sufficient number of figures for the number of decimals required, annex ciphers to fill the deficiency, and proceed according to the rule.

If, for instance, seven places of decimals were required in the last example, instead of five, the line would be drawn two places farther to the right (filling up with ciphers, or dots to represent them); then, annexing two more dots to the right, so as to make the correct number (five) to the right of the line, proceed as before.

METHODS OF PROOF.

There are two principal methods of proving Division:

First, by multiplication: Multiply the quotient by the divisor, and to the product add the remainder, if any; the result, if the work is correct, will be equal to the dividend.

Second, by casting out the 9's. This method of proof, which is very easy and convenient in practice, and generally preferred by the experienced arithmetician, is given in connection with the following:

EXAM. How many times is 83 contained in 2869086?

2	2869	086	÷ 83	3
	34429	032	÷ 996	
	137	716		
		548		
		4		
7	34567	300	÷ 12 = 25 rem.	3

To divide by 83, we multiply both dividend and divisor by 12, and the new divisor is 996, to which Rule III is applicable. The quotient is 34567, and the remainder, 300, which is divided by 12, to get the true remainder, 25.

PROOF. Commencing with the divisor, 83, we add its digits from right to left, rejecting 9 from the sum; thus: 3 and 8 are 11; 9 from 11 and 2; the remainder, or excess, 2, is now carried to the left and reserved.

The digits of the quotient are next added, and 9 rejected, in like manner; thus: 7 and 6 are 13, 9 from 13 and 4; then this 4 and 5 are 9, and taking 9 from this, leaves 0; next, 4 and 3 are 7, from which 9 cannot be taken; 7 is carried to the left, as shown in the margin.

Multiplying the excesses, 7 and 2, now, gives 14; the digits of this are added, in like manner, making 5, which, being less than 9, is added to the digits of the remainder, 25, from left to right; thus, 5 and 2 are 7, and 5 are 12; then, 9 from 12 and 3, the excess, is carried to the right and reserved.

Finally, adding the digits of the dividend, from left to right, also; omitting 9 whenever it occurs, and rejecting 9 from the sums, as often as they make 9, or more, we get an excess of 3, which is equal to the excess found from the remainder, and the work is supposed to be correct.

RULE. (1) *Cast the 9's from the divisor and quotient; set the excesses to the left of the work and reserve them.* (2) *Multiply said excesses, and cast the 9's from the product; add the excess here found,*

to the remainder, and cast the 9's from the sum, reserving the excess to the right of the work. (3) *Cast the 9's from the dividend, setting the excess to the right, also; if both excesses on the right be equal, the work is presumed to be correct.*

NOTES.— 1. Should the excess from the divisor be 0, it is evident we need not go over the quotient, as the result found by multiplication would also be 0. The excess from the remainder, if any, in that case, will be the same as that from the dividend; and if there be no remainder, the excess from the dividend must be 0.

2. The divisor, 996, and its corresponding dividend, might be taken in proving the work in the foregoing example, but in that case, the corresponding remainder, 300, must be taken, instead of 25, 300 being the true remainder for 996. Try it, the excess will be 0.

3. It may be well to remark, also, that when the division is continued into decimals, *the true remainder*, and *not the decimal*, must always be taken when using this method of proof.

4. It is hardly necessary to say that, should there be a misplacing of figures (an occurrence, however, which is very rare), this method of proof will fail, as it is evident the *sums* of the digits will be the same, regardless of their local positions.

When the division is performed by the simplified methods, the proof can be obtained by the short methods for multiplication commencing at page 71; as illustrated in the two following examples:

EXAM. 1. In 497736 square inches, how many square feet?

To divide by 144 (sq. inches in a sq. foot) both numbers are multiplied by 7, giving 3484152÷1008.

The quotient is 3456, and the remainder, 504.

To prove the work, 3456 is multiplied by 1008, and the remainder, 504, is added. This is performed by simply setting 8 times 3456, or 27648, three places to the right, under the remainder, and adding; this gives 3484152, the dividend. (*See example* 2, *page* 71.)

```
3484|152 ÷ 1008
  27|872
----|---
3456|280
    |224
    |---
    |504
   27648
 -------
 3484152
```

EXAM. 2. Divide 3174473 by 497.

Doubling both numbers, we have 6348946 to be divided by 994, a simple divisor. The quotient is 6387, and the remainder, 268.

To prove the work, 6387 is multiplied by 994, and 268 added. The process is performed by setting 6 times 6387, or 38322, three places to the right, under the remainder, and subtracting; this gives the dividend. (*See example* 3, *page* 72.)

6348	946 ÷ 994
38	088
	234
6387	268
38	322
6348946	

NOTE.—The proof is to be always taken before reducing the remainder to a decimal.

ON SIMPLE DIVISORS.

Divisors may be simplified by any process that will make them 10, 100, 1000, 10000, etc., or that will make them a little more, or a little less, than these; as, 101, 102, 1003, 10007, etc.; or 91, 92, 98, etc.; 991, 992, 993, 9989, 9999, etc.

NOTE.—Composite numbers, when not too large, can be readily divided by using their component factors. This method is called successive division. A composite number is one that may be produced by multiplying together two or more numbers. Thus: 18 is equal to 6×3; or 9×2; or $3 \times 3 \times 2$.

EXAM. In 12872 cubic feet of earth, how many cubic yards, or loads?

In this, the component factors of 27 (cub. feet in a cub. yd.) are 3 and 9 ($3 \times 9 = 27$). We divide first, by 3, and the result by 9, to get 476.74 cubic yards, or loads.

3	12872
9	4290.66′
	476.74

NOTE.—*In making calculations where the divisor is constant, and is a composite number*, the desired results are more easily obtained by successive division than by the usual long methods; thus: 2240 pounds to a gross ton, the factors are 40, 7 and 8 ($40 \times 7 \times 8 = 2240$). Again, 5280 feet to a mile; the factors are 60, 8 and 11 ($60 \times 8 \times 11 = 5280$); and 43560 square feet to an acre; the factors are 40, 11 and 99 ($40 \times 11 \times 99 = 43560$); and by the short methods already established, divison by 99 is the simplest part of the process. (*See page* 55.)

The following suggestions will aid the student in obtaining simple divisors :

To divide by

$11\frac{1}{2}$: 11.5, or 115, multiply by 8; the results are 92 and 920, simple divisors.

$12\frac{1}{2}$: 12.5, 1 25, 125 or 1250, multiplied by 8 will give 100, 10, 1000 and 10000.

13: $13\frac{1}{4}$, 13.25, 1.325 or 1325, multiplied by 8 will give simple divisors.

$13\frac{1}{2}$: 13.5, 1.35, 135 or 1350, multiplied by 8 will give simple divisors.

14: The component factors are 2 and 7; or multiply by 7 the result is 98.

$14\frac{1}{4}$: 14.25, 142.5, 1425 or 14250 multiplied by 7 give simple divisors.

$14\frac{1}{2}$: 14.5, 145, 1450, etc., multiplied by 7 give simple divisors.

To divide by $14\frac{1}{4}$ or 14.25; multiply by 7, the result is $99\frac{3}{4}$; use 100 for the approximate divisor, and add for the quarter, etc.

EXAM. Divide 109185 by 145.

In this, both numbers are multiplied by 7, and we have 764295÷1015. But a slight inspection shows that this can be still further simplified. By setting 145 one place to the right, under 1015; and the given dividend one place to the right under the new dividend, and subtracting in each case, we get 10005 for a simple divisor. The required quotient is 753.

```
109185 ÷  145
764295   1015
 109185    145
753|3765÷10005
   |3765
```

$16\frac{1}{2}$: 16.5, 165, 1650, 16.6, 166, 1666, etc.; 167, 16.7, 1670, 1675; 168, 16.8, 1680, etc.; and $16\frac{3}{4}$, all multiplied by 6 give simple divisors; thus: $16\frac{1}{2} \times 6 = 99$; etc.

17: Multiplied by 6 gives 102; and $17\frac{1}{4}$, or $17.25 \times 4 = 69$. (*See pages* 250 *and* 251.)

$17\frac{1}{2}$: 17.5, 175, 1750, etc.; multiplied by 6 give simple divisors.

$17\frac{3}{4}$: 17.75, 177.5, 1775, 17750, etc.; multiplied by 8×7 give simple divisors. Thus: $17\frac{3}{4}$ or $17.75 \times 8 = 142$, and $142 \times 7 = 994$, a simple divisor.

18: The factors are 3 and 6; or $18 \times 6 = 108$, a simple divisor.

$18\frac{1}{4}$: 18.25, 1825, 18250, etc.; multiplied by 4 give 73, 730, 7300, 73000, etc.; and the method of dividing by these is given at pages 52 and 53.

$18\frac{3}{4}$: 18.75, 1875, 18750, etc.; multiply by 4 and to the result add $\frac{1}{3}$ of itself. Thus: $18\frac{3}{4} \times 4 = 75$; $\frac{1}{3}$ of which is 25, and this added to 75 gives 100; etc.

19: Multiplied by 5; the result is 95: $19\frac{1}{4}$, 19.25 or 1925 multiplied by 5 and a fifth of the number itself added; thus: $1925 \times 5 = 9625 + \frac{1}{5}$ of $1925 = 10010$.

It must be carefully borne in mind that, whatever change is made in the divisor, to simplify it, a similar change must be made in the dividend to preserve the relation; and that when the divisor can be simplified, any multiple, sub-multiple, or aliquot part of it can also be simplified. (*See page* 250.)

And now, if the numbers from 19 to 100, be taken and examined, the subject will be found not only interesting and instructive but it will be found that division by the majority of these, their multiples and sub-multiples, will be exceedingly simple. For instance, most of the twenties multiplied by 4; the thirties by 3; the forties and fifties by 2, etc., will give simple divisors.

A few more examples of a practical nature, before closing this chapter, may be found helpful.

EXAM. In 1269 pounds of oil; how many gallons, $7\frac{1}{2}$ pounds to the gallon?

In this, we simply add a third; and a tenth of the result is the number of gallons, and the decimal of a gallon. In other words, by adding a third to $7\frac{1}{2}$ it becomes 10, a simple divisor.

$$\begin{array}{rr} 1296 \div & 7\frac{1}{2} \\ 432 & 2\frac{1}{2} \\ \hline 172.8 \div & 10 \end{array}$$

EXAM. In 12134 feet; how many perches of $16\frac{1}{2}$ feet?

Here, both numbers are multiplied by 6, and we have $72804 \div 99$. The answer is $735\frac{39}{99}$, or reducing the fraction to a decimal, .3939, etc.

$$\begin{array}{r|l} \multicolumn{2}{r}{12134 \div 16\frac{1}{2}} \\ \hline 728 & 04 \div 99 \\ 7 & 28 \\ & 7 \\ \hline 735 & 39\ . \end{array}$$

EXAM. In 34682487 feet; how many miles of 5280 feet?

We give the solution of this two ways:

First: By using the factors $(60 \times 8 \times 11 = 5280)$. Cutting off one figure, and dividing by 6, divides by 60. Then an eighth of this, and one eleventh of the eighth.

$$\begin{array}{r|l} 3468248 & 7 \\ 578041 & 45 \\ 72255 & 1812 \\ \hline 6568 & 6528 \end{array}$$

Second: Here, we take the component factors, 80, 11 and 6, or 80 and 66 ($66 \times 80 = 5280$) and divide, first, by 80, by cutting off one figure and dividing by 8. Then, to divide by 66, we add half to get 99, a simple divisor. The remainder of the process is clear.

The answer is 6568.6528 miles. The decimal may be carried to any desired length, by annexing ciphers to the remainder and dividing by 99.

```
 3468248.7
 ---------
  433531.0875÷66
  216765.5437  33
 ---------------  --
 6502|96.6312÷99
   65|02
     |66
 --------------
 6568|6463|12
     |  64|63
     |    |64
     -----
    .6528|39
```

EXAM. In 76824763 square feet; how many acres, true to five places of decimals, 43560 sq. ft. to the acre?

The component factors of the divisor, in this, are 40, 11 and 99 ($40 \times 11 \times 99 = 43560$). The solution is left for the student. Should he fail to see his way clear, he is referred to page 55, where the explanation is given.

And when the digits follow in their natural order, the division can be simplified, as illustrated in the following:

EXAM. Divide 1524157875019052I by 123456789.

```
    15241578750190521 ÷  123456789
   121932630001524168     987654312
  --------------------   ----------
  123456787|87654322|01 ÷ 99999999|09
          1|23456787
           |       2
  ---------------------
  123456789|1111111101
           |1111111101
           ------------
```

Here, we first multiply both dividend and divisor by 8, setting the product under each respectively, one place to the left of units, and adding, in both cases, we take the results for a new dividend

and new divisor. Cutting off 09 from the divisor, now, and 01 from the dividend, we divide the remaining part of the dividend by the remaining part (99999999) of the divisor, 1 being the key, and we get 123456789 for quotient, and 11111111 for remainder, to which 01, cut from the dividend, is annexed, and the remainder is then 1111111101.

The result thus obtained is too large, being the quotient for 9999999900; to correct, we subtract 9 times (09 cut from the divisor) the quotient, 123456789, or 1111111101. There is no remainder, 123456789 being the quotient.

NOTES.—1. Multiplying the terms by 8, setting the products one place to the left, and adding, we need scarcely observe, is multiplying by 81.

2. The complement of 9999999909 being 91, the quotient may be obtained without cutting off the figures as was done in the example, by adding 91 times the quotient. In that case there would be no subtraction. And on examination, it will be found that is what we have actually done, because to multiply by 91, we have used the short method, namely, multiplied by 100, and subtracted 9 ($100 - 9 = 91$). Hence, the

RULE. *To divide by the nine digits in their natural order:*

Set 8 times the dividend under itself one place to the left, and divide the sum by 9999999909, or by eight 9's, 09.

And when the digits are in a reversed order, we would proceed as in the following:

EXAM. Divide 975461057789971041 by 987654321.

```
     975461057789971041 ÷    987654321
   7803688462319768328      7901234568
8)7901234568|0987654321 ÷ 80000000001
   987654321|0123456790125
            |0123456790125
```

In this, we multiply the terms by 8, setting the products one place to the left, as in the last example, and adding, the new divisor is 80000000001, the key being $\frac{1}{8}$.

Dividing, first, by 80000000000 (simply cut off ten figures from the right of the dividend, by the vertical line, for the ten ciphers, and divide by 8), we get 987654321 for quotient and |01234567890125 for remainder. From the result thus found we subtract $\frac{1}{8}$ of the quotient, set in proper position; that is, 1-80000000000 part of 987654321, or .01234567890125. There is no remainder, 987654321 being the required quotient. Hence, the

RULE. *To divide by the nine digits written in reversed order:* Set 8 times the dividend under itself one place to the left, and divide the sum by 80000000001, or by 8 followed by nine ciphers and 1.

NOTE. — To get 1-80000000000 of the quotient, 987654321, is to divide the latter by 80000000000, and to do so we simply cut off the ten ciphers and divide by 8. But we find that the number to be divided (the quotient) does not contain ten figures to be cut off, to correspond with the number of ciphers, so we prefix a cipher to supply the deficiency; the number to be divided then is .0987654321, and the eighth part of this is .01234567890125, which is equal to the remainder from which it is subtracted, giving 0 for remainder (all of which can be seen at a glance after a little practice with our methods).

MIXED NUMBERS.

A Mixed Number consists of a whole number with a fraction annexed; as, $34\frac{1}{8}$, $106\frac{7}{8}$, $249\frac{1}{4}$, etc., and division by these will be found as simple as whole numbers; thus:

To divide by $249\frac{1}{4}$, *or its equal,* 249.25: *Multiply by* 4 *and the new divisor is* 997, *a simple divisor* (not forgetting to multiply the dividend, also, by 4).

$251\frac{1}{2}$, or its equal, 251.50: Multiplied by 4, gives 1006 for a simple divisor.

$167\frac{5}{6}$, or its equal, 167.8333, etc. (3 being repeated), which may be written $167.8\frac{1}{3}$: Multiplied by 6, gives 1007 for a simple divisor; and so of other mixed numbers.

SHORT METHODS FOR MULTIPLICATION.

Before entering into the more important of our short methods for Multiplication, we deem it proper to give, at the commencement, a few of the more simple, with which, no doubt, some of our readers are already familiar, but, a knowledge of them is essential to all, to fully understand the several cases which follow.

To multiply by 1, followed by any number of ciphers:

Simply annex to the multiplicand as many ciphers as there are in the multiplier.

Exam. 1. Multiply 475891 by 1000.

475891000

Here, there are three ciphers in the multiplier, and we simply annex three ciphers to the multiplicand to find the product, 475891000.

Exam. 2. Multiply 475891 by 1007.

$$\begin{array}{r} 475891\ldots \\ 3331237 \\ \hline 479222237 \end{array}$$

Here, we first multiply by 1000, as in example 1, using periods, or dots, instead of ciphers, and to the product add 7 times the multiplicand.

EXAM. 3. Multiply 475891 by 993.

$$\begin{array}{r} 475891\ldots \\ 3331237 \\ \hline 472559763 \end{array}$$

Here, we first multiply by 1000, as in the two previous examples, and from the product subtract 7 times (1000 — 993) the multiplicand.

To multiply by 21, 31, 41, etc.: *Simply set the product by the tens under the multiplicand, in proper position, and add;* thus:

EXAM. 4. Multiply 6853 by 71.

$$\begin{array}{r} 6853 \times 71 \\ 47971 \\ \hline 482563 \end{array}$$

And if ciphers come between the two digits of the multiplier, proceed in the same way, only move the product as many places to the left as there are figures in the multiplier; thus:

$$\begin{array}{r} 6853 \times 7001 \\ 47971\ldots \\ \hline 47977853 \end{array}$$

Composite numbers which can be readily factored should always be used, so as not to need addition in the multiplication; thus:

EXAM. 5. Multiply 3684 by 42.

$$\begin{array}{r} 3684 \times 42 \\ \hline 22104 \\ \hline 154728 \end{array}$$

Instead of first multiplying by 2 and next by 4, and adding the partial products, we prefer to use the component factors 6 and 7 ($6 \times 7 = 42$). Multiplying first by 6, we get 22104, and multiplying this in turn by 7, we have the product, 154728, without addition.

As we shall make use of subtraction to a large extent in the short methods for Multiplication, we may be permitted here to make a slight digression to say a few words on

SUBTRACTION.

The usual method in Subtraction is to place the less number below the greater, with units under units, etc.; but it will be often found of greater advantage to reverse this order, by having the less placed *above* the greater; or, by having both the less and the greater contained in *one number*. Both the latter methods will be used in the following short methods for Multiplication, but in the one case the words *upper* and *lower* will be interchanged throughout the rule, and in the other we shall point out the two numbers when contained in one.

EXAM. 6. Multiply 6847 by 4193.

```
          42..
     6847 × 4193
    ------------
     47929     7
   287574..
   --------
   28709471
```

Here, instead of multiplying by 4193 and adding the four partial products, we take 4200 for approximate multiplier. The difference of the multipliers is 7, by which we first multiply, getting 47929. Now, we see that 7 is one of the factors of 42; dividing

42 by 7 gives the other factor, 6. It is evident, now, that by multiplying the product of 7, or 47929, by 6, we get the product for 42, or 287574, and that by simply moving this number three places to the left of units, it represents the product of 4200. Then subtracting the *upper* number from the *lower*, in other words, taking the product of 7 from that of 4200, gives the product for 4193 (4200 — 7 = 4193).

And if the multiplier were 41993, 419993, etc.; 3493, 34993, etc.; 3495, 34995, etc., we would proceed in a similar manner. using 42000, 420000, etc.; 3500, 35000, etc., for approximates; and so on with other numbers of a like nature.

EXAM. 7. Multiply 4376 by 999.

$$\begin{array}{l} 4376\ldots \times 999 \\ \hline 4371624 \end{array}$$

Here, we multiply by 1000, by simply conceiving three ciphers, represented by dots, annexed; the result is 4376000, or rather, 4376..., as seen in the margin. Now, if from this we take once the multiplicand, 4376, the difference will be the product of 999 (1000 — 1 = 999). It will now be observed that the product of 1000, and the product of 1, are both contained in the one expression, 4376..., and that by simply taking 4376, the product of 1, from the whole, 4376..., the product of 1000, the difference is the required result; the subtraction being performed without setting down 4376 a second time.

REMARK.— It may, perhaps, be well to remark, that it is immaterial in Multiplication which factor is taken as multiplier, or which as multiplicand; or what position the partial products hold with reference to the multiplicand, whether they be placed to the right or the left; provided they are assigned the proper position with respect to each other.

EXAM. 8. Multiply 3984 by 3476.

$$\begin{array}{rcr}
4\ldots & & \\
\underline{3984} & \times & \underline{3476} \\
16 & & 13904\ldots \\
 & & \underline{55616} \\
 & & 13848384
\end{array}$$

A short inspection of the factors here, shows that by taking the multiplicand, 3984, as the multiplier, the process can be shortened. Taking 4000 for approximate, the difference of the multipliers is 16. Multiplying 3476 by 4, and annexing three ciphers (dots) gives the product of 4000. Now, we observe that 4, the significant figure of 4000, is contained 4 times in 16, the difference of the multipliers, and that by multiplying 13904, the product of 4, by 4 (found by dividing 16 by 4), and setting the result, 55616, in proper position, it represents the product of 16. Subtracting this, now, from 13904...; that is, taking the product of 16 from that of 4000, gives the product for 3984 (4000 — 16 = 3984).

To multiply by any number from **11** *to* **19.**

EXAM. 9. Multiply 743586 by 11.

$$\begin{array}{r}
\underline{743586 \times 11} \\
8179446
\end{array}$$

Here, we simply set down first the unit figure, 6, of the multiplicand, then add the figures from right to left, carrying when necessary as we proceed, thus: 6 and 8 are 14; 8 and 5, and 1 carried, are 14; 5 and 3, and 1 carried, are 9; 3 and 4 are 7; 4 and 7 are 11; 7 and 1 carried are 8.

EXAM. 10. Multiply 7468 by 17.

$$\begin{array}{r}
\underline{7468 \times 17} \\
126956
\end{array}$$

Here, we multiply by 7, the unit figure of 17, adding the figures of the multiplicand as we proceed, thus: 7 times 8 are 56; 7 times 6 are 42, and 8 (the unit figure) and 5 (carried) are 55; 7 times 4 are 28, and 6 (the figure to the right of 4) and 5 (carried) are 39; 7 times 7: 49 and 4, and 3 (carried) are 56; 7 (the last figure) and 5 (carried) are 12.

NOTE. — In such cases it is better to always add the figure of the multiplicand first, adding in the figure carried after.

EXAM. 11. Multiply 7258 by 1013.

$$\begin{array}{r l} 7258\ldots & \times\ 1013 \\ 94354 & \\ \hline 7352354 & \end{array}$$

Here, we first multiply by 1000, and next by 13 (short method), and add.

EXAM. 12. Multiply 13986 by 3684

$$\begin{array}{r r} & 14\ldots \\ 3684\ \times & 13986 \\ \hline 51576\ldots & 14 \\ \hline 51524424 & \end{array}$$

Taking the multiplicand in this (seeing that it is near 14000) for the multiplier, the process can be shortened at once. Multiplying 3684 by 14 (short method), and then annexing three ciphers (dots) we have the product of 14000, or 51576... Now, the difference of the multipliers is also 14, and it is evident that if 14 times the multiplicand, 3684, be taken from the product of 14000, the difference will be the product of 13986. Now, the product of 14, and also of 14000, are contained in the one expression, 51576...; all we have to do, then, is to subtract 51576 from 51576..., and the difference is 51524424, the required product. (See example 7.)

EXAM. 13. Multiply 35982 by 3286.

```
36...
35982 × 3286
──────────────
    18  59148
       ──────
       118296...
       ─────────
       118236852
```

Here, we see tha. 35982 is near 36000, so we take these two as the multipliers. Their difference is 18. Multiplying 3286 by 18 (short method), the product is 59148. Multiplying this by 2 gives the product of 36 ($18 \times 2 = 36$); annexing three ciphers (dots) gives the product of 36000, or 118296... Now, if from 36000 we take 18, the difference is 35982. Subtracting 59148, the product of 18, from 118296..., the product of 36000, gives 118236852, the required product.

EXAM. 14. Multiply 46782 by 27985.

```
                28...
       46782 × 27985
      654948   ─────
                  15
  ──────────
  1309896...
  ──────────
  1309194270
```

If the difference of the multipliers, in this example, was 14 instead of 15, we could proceed as in the last. Here, we set 14 times (short method) the multiplicand under itself, and we see at a glance that if both numbers be added the result is the product of 15 ($14 + 1$). We do not add, however, but set 2 times 654948, the product of 14, four places to the left, having first drawn a separating line, and conceiving three ciphers annexed, we have the product of 28000, or 1309896... Now, if from this product, the sum of the two numbers immediately above it be taken, the difference will be the product of 27985 ($28000 - 14 + 1$).

NOTE.—It is hardly necessary to say to the arithmetical student, that, in taking the two partial products which go to make up the product of 15, from that of 28000, the addition and subtraction go hand in hand, thus: Commencing at the top; 2 and 8 are 10, from 10, and 0. Carry 1 to 8, 9 and 4 are 13, from 20 (in this case) and 7; 2 and 7, 9, and 9 are 18, from 20 and 2; 2 and 6, 8, and 4 are 12, from 16 and 4, etc.

EXAM. 15. Multiply 24975 by 5437.

$$\begin{array}{r} 24975 \times \overline{5437}.. \\ \overline{135925}... \\ \overline{135789075} \end{array}$$

Seeing that 24975 is near 25000, we take these two numbers for multipliers; we see by inspection that the difference is 25. We multiply by 25000, thus: Conceiving two ciphers annexed to 5437 multiplies that number by 100, and taking one-fourth of the result gives the product of 25, or 135925. To this we annex three ciphers (dots), and we get 135925..., the product of 25000. Now, taking 25 from 25000, gives 24975, that is, 135925... minus 135925 (both contained in the one number), gives 135789075, the required product.

When the quotient obtained by dividing the difference of the multipliers by the significant part of the approximate multiplier, consists of two or more figures which can be readily factored, we would proceed as in the following:

EXAM. 16. Multiply 59832 by 78431.

$$\begin{array}{lll} 6.... & & \\ \underline{59832} & \times & \underline{78431} \\ \quad \frac{168}{6} = 28 & & 470586....1882344 \\ & & \quad 13176408 \\ & & \overline{4692683592} \end{array}$$

Taking 59832 and 60000 for the multipliers, in this example, we find their difference to be 168, which contains 6, the signifi-

cant part of 60000, 28 times. Multiplying 78431 first by 6, and annexing four ciphers (dots) we have the product of 60000. Now, 28 times 6 is 168, and if 168 be taken from 60000 it leaves 59832; in other words, if 168 times 78431 be taken from 60000 times that number, or 470586...., which we already have, the difference will be the product for 59832. To get 168 times the multiplicand, 78431, we take 28 times the product of 6. To multiply by 28, we use its component factors, 4 and 7 (4 × 7 = 28), setting 4 times 470586 a little to the right, as shown in the margin; then 7 times that product, or 13176408, is set under the product of 60000 and subtracted, giving 4692683592, the required product.

When one part of the multiplier, or of the multiplicand, is a multiple of another part, illustrated in the following:

EXAM. 17. Multiply 47634 by 16128.

```
  47634 × 16128
 762144...
   6097152
 768241152
```

A glance at the multiplier, 16128, in this example, shows that 128, the three last figures, is 8 times 16, the two first, in other words, 128 is a multiple of 16.

Multiplying first by 16 (short method), and conceiving three ciphers annexed to the result, gives the product of 16000; then, setting 8 times 762144 in proper position, and adding, we get the required product.

NOTE.—If the multiplier or the multiplicand had been 12816, we would proceed in the same way, first multiplying by 16, as in the example; then setting 8 times the product of 16, three places to the left and adding. And if one of the factors were 1612832, 3212816, 1283216 or 1632128, the process would be equally simple; multiplying first by 16, then 8 times the product

of 16, and next, 2 times the product of 16; taking care to place the partial products in proper position with respect to one another. And so of other numbers similarly combined; such, for instance, as 312, 3012, 315, 3015, etc.; 412, 416, 424, 4024, etc.; 642, 6042, 756, 7056, etc., etc.

DIVISION REVERSED.

Many extraordinary contractions in Multiplication may be obtained by simply *reversing* our new methods for Division. A few examples will illustrate.

EXAM. 1. Multiply 756 by 334.

$$\begin{array}{r} 756\ldots \\ 1512 \\ \hline 252504 \end{array}$$

Here, we simply annex three ciphers (dots) to 756, and under the result set 2 times 756, or 1512. The sum of these two numbers, without actually setting down the result, is evidently 757512, and we divide both numbers, considered as one, by 3, getting 252504, the required product.

Reason: To divide by 334, we would multiply by 3, getting 1002; divide first by 1000 and subtract 2 times the quotient. In the multiplication we *reverse* the process, multiplying first by 1000, adding 2 times the multiplicand, and then dividing by 3.

And if one or both of the factors were 3334, 33334, etc., the process would be equally simple.

EXAM. 2. Multiply 1668 by 478.

$$\begin{array}{r} 4783824 \\ \hline 797304 \end{array}$$

Here, we simply set 8 times 478, or 3824, to the right and divide by 6, and we have 797304, the product.

Reason: To divide by 1668, we would multiply it by 6 (being nearly the sixth of 10000), getting 10008 for new divisor.

To divide by 10008, we first divide by 10000, then subtract 8 times the quotient. To multiply by 1668 (which in the example we have made the multiplier), we reverse the division by first multiplying by 10000, adding 8 times the multiplicand, and then dividing by 6.

And if one or both factors were 16668, 166668, etc., the process would be equally simple.

EXAM. 3. Multiply 7342 by 8334.

$$\begin{array}{r} 7342\ldots\ldots \\ 58736 \\ \hline 61188228 \end{array}$$

Setting 8 times the multiplicand, or 58736, five places to the right, in this, and looking on both numbers as one (that is, as being added), we divide by 12 to get 61188228, the required product.

Reason: To divide by 8334, we multiply it by 12, getting 100008 for divisor; reversing the division, we have the multiplication.

EXAM. 4. Multiply 142858 by 37684.

$$\left.\begin{array}{r} 142858 \\ \hline 1000006 \end{array}\right\} \begin{array}{r} 37684\ldots\ldots \\ 226104 \\ \hline 5383460872 \end{array}$$

In this we take 142858, as the multiplier: To divide by this number we multiply it by 7, getting 1000006 for divisor, as shown in the margin.

To multiply by 142858, we reverse the division by multiplying 37684 by 1000000, adding 6 times 37684, or 226104, and dividing by 7.

And the numbers 1428, 1429, 14286, 14287, 142857, etc., will be found equally simple, and so with other numbers.

EXAM. 5. Multiply 123456789 by 123456789.

```
      123456789..........
                11234567799
   9)12345678787654322O1
    9)1371742087517146 89
       152415787501905 21
```

In this we simply conceive ten ciphers annexed to the multiplicand, and from the result we take 91 times (short method) the multiplicand, or 11234567799; then dividing the difference by 9 and 9, in succession, we have the required product.

This is the reverse of division by the nine digits in direct order. (See example, page 68.)

RULE. *To multiply any number by the nine digits in direct order:* Annex, or conceive to be annexed, ten ciphers to the number to be multiplied; from the result take 91 times the said number and divide the difference by 9 and 9 in succession.

EXAM. 6. Multiply 987654321 by 987654321.

```
     987654321..........
   79012345680987654321
     8779149520109739369
       975461057789971041
```

Conceiving ten ciphers annexed to the multiplicand, in this; multiplying the result by 8, and adding the multiplicand, we divide the sum by 9 and 9, in succession, as in the preceding example, and we have the required product.

This is the reverse of division by the nine digits written in reversed order (see example, page 69).

RULE. *To multiply any number by the nine digits in a reversed order:* Annex, or conceive to be annexed, ten ciphers to the multiplicand; to 8 times the result add the multiplicand, and divide the sum by 9 and 9 in succession.

NOTE.— An unlimited number of such examples could be added, but, for the student who has carefully read the preceding pages of this work, enough has been given by way of suggestion.

To square any number of two digits we give the following simple

RULE. (1) Add to the given number the difference between it and the next higher number ending with a cipher. (2) Subtract from the given number the number which was added. (3) Multiply the sum and difference thus found, and to the product add the square of the figure added.

EXAM. 1. What is the square of 47?

First, add 3, making	50
Second, subtract 3, making	44
Their product is.........	2200
Now add 3 times 3, or ...	9
	2209, the square.

Or, if more convenient, subtract from the given number the difference between it and the next *lower* number ending with a cipher; then add to the given number that which was subtracted, and proceed as above, thus:

EXAM. 2. What is the square of 91?

First, subtract 1, making..	90
Second, add 1, making ...	92
Their product is.........	8280
Add 1 × 1, or...........	1
	8281, the square.

The process is founded on the principle that: *The product of the sum and difference of two numbers, is equal to the difference of their squares:*

If $47 + 3$ be multiplied by $47 - 3$, the product is equal to 47 squared minus 3 squared; that is,

$$(47 + 3) \times (47 - 3) = 47^2 - 3^2$$

Now, $47 + 3$ is the same as 50, and $47 - 3$, the same as 44; therefore 50×44 is equal to (47×47) minus (3×3), that is, when 50 is multiplied by 44, and the square of 3, or 9 added to the product, the result will be the square of 47; thus:

$$50 \times 44 = 2200; \text{ and } 47^2 - 3^2 = 2209 - 9; \text{ that is,}$$
$$2200 = 2209 - 9$$

Now, it is a well-known axiom that: *if equals be added to equals, the sums will be equal.* If, now, 9 be added to the two last expressions, which are equals, the sums will be equal, thus: $2200 + 9 = 2209 - 9 + 9$; subtracting 9 and adding it at the same time leaves the last expression 2209; and adding 9 to 2200 makes it 2209, which is the square of 47.

NOTE.—A little practice will enable a person to readily square any number consisting of two digits mentally by this simple method.

This method will be found convenient in many cases where the number to be squared consists of three figures.

EXAM. 3. What is the square of 109?

$$109^2 = 100 \times 118 + 81 = 11881, \text{ at sight.}$$

Numbers consisting of three figures may be readily squared on the same principle, thus:

EXAM. 4. What is the square of 432?

$$432^2 = \begin{cases} 400 \times 464 = & 185600 \\ 30 \times \ \ 34 = & 1020 \\ 2 \times \ \ \ 2 = & 4 \end{cases}$$
$$\overline{186624}$$

Subtracting 32 gives 400, adding 32 gives 464; the results are multiplied, giving 185600. The square of 32 is added next; subtracting 2 gives 30, adding 2 gives 34; the results are multiplied, giving 1020. Finally, the square of 2 is added, making 186624, the square of 432.

EXAM. 5. What is the square of 371?

$$371^2 = \begin{cases} 442 \times 300 = & 132600 \\ 72 \times \ \ 70 = & 5040 \\ 1 \times \ \ \ 1 = & 1 \end{cases}$$
$$\overline{137641}$$

Adding 71 makes 442, subtracting 71 makes 300; their product is 132600.

Adding 1 to 71 makes 72, subtracting 1 makes 70; their product is 5040.

Adding the square of 1 gives 137641, the square of 371.

RULE. *To square any number of two figures:* (1) *Multiply the units by the units and set down the unit figure of the product.* (2) *Multiply the sum of the units by a single figure of the tens, set down the unit figure of the product.* (3) *Multiply the tens together to complete the square, carrying as usual.*

EXAM. What is the square of 74?

Ans. $74^2 = 74 \times 74 = 5476$.

Here, say 4 times 4 are 16; set down 6, and carry 1; then, 7 times 8 (4 + 4) are 56 and 1 are 57; set down 7, and carry 5: next, 7 times 7 are 49, and 5 are 54 completes the square. (*See page* 271.)

RULE. *To square numbers of two figures ending in* 5 *:* (1) *Multiply the* 5*'s together and set down the result in full.* (2) *Add* 1 *to either figure of the tens and multiply the other by the number thus increased.*

EXAM. What is the square of 75?

Ans. $75^2 = 75 + 75 = 5625$.

Say 5 times 5 are 25; set down in full; add 1 to either 7 and say 8 times 7 are 56 to complete the square.

Reason : $75 \times 75 = 80 \times 70$ plus $5 \times 5 = 5600 + 25 = 5625$.

The rule is applicable in many cases to three, four or more figures; thus: 125; 135, 145, etc., 295; 395; 39995, etc.

EXAM. What is the square of 695?

Ans. $695^2 = 695 \times 695 = 483025$.

In this say 5 times 5 are 25; set down in full; add 1 to 69 and say 70 times 69 are 4830 to complete the square. And so with the others and similar numbers.

And the same rule can be applied to any two figures, whose units when added make 10; *the tens in both numbers being alike;* as 47×43; 76×74; 58×52, etc., and in many cases to three, four, or more figures, as 172×178; 127×123; 196×194; 193×197; etc., 292×298, 293×297; 394×396; 4993×4997; etc.

EXAM. Multiply 76 by 74. Ans. $76 \times 74 = 5624$. Say 4 times 6 are 24; set down in full; add 1 to either 7, and say 8 times 7 are 56; for the *reason* that $76 \times 74 = 80 \times 70$ plus $6 \times 4 = 5600 + 24 = 5624$.

EXAM. Multiply 397 by 393. Ans. $397 \times 393 = 156021$. Say 3 times 7 are 21; set down in full; add 1 to 39 (either factor) and say 40 times 39 are 1560 to complete the product. (*See examples on page* 87.)

NOTE.—When the units are 1 and 9 the second figure of the product will always be a cipher; as: $71 \times 79 = 5609$, $691 \times 699 = 483009$. Say 9 times 1 are 9, then 0, 8 times 7 are 56. Say 9 times 1 are 9; then 0; and 70 times 69 are 4830 completes the work.

EXAM. Multiply 127 by 124. Ans. $127 \times 123 = 15621$

Plus 127 multiplied by 1 = 127

$$\begin{array}{r} 15621 \\ 127 \\ \hline 15748 \end{array}$$

NOTE.—Multiply as if the units made 10, and add or subtract for the difference.

Hence, in all cases where the two units figures make 10, *and the other figures are alike*, the foregoing rule is applicable, as illustrated in the following

EXAMPLES:

$127 \times 123 = 15621$:	Here, because 3 and 7, the unit figures, make
112×118	10, and 12 and 12 are alike, we simply say 3
47×43	times 7 are 21, setting it down in full; then,
84×86	adding 1 to either 12, calling it 13, we say 12
134×136	times 13 are 156, which completes the product,
etc.	15621.

And if one or both factors contain a fraction, the process will be found equally simple; thus:

$24 \times 26\frac{1}{2} = 636$:	In this, 6 and 4 make 10, and the 2's are alike;
$28\frac{1}{2} \times 22$	say one half of 24 is 12, to carry: then, 6 times 4
$36 \times 34\frac{1}{4}$	are 24, and 12 are 36; set down in full; now add 1
$48 \times 42\frac{1}{8}$	to 2 and say 3 times 2 are 6; this completes the
etc.	product.

$24\frac{1}{2} \times 26\frac{1}{4} = 643\frac{1}{8}$:	Here, we say $\frac{1}{4}$ of 24 is 6, to carry: then, 6
$12\frac{1}{2} \times 18\frac{1}{4}$	times 4 are 24, and 6 are 30; now, 3 times 2 are
$32\frac{1}{4} \times 38\frac{1}{2}$	6; making 630, or $24 \times 26\frac{1}{4}$: to this is added $\frac{1}{2}$
etc.	of $26\frac{1}{4}$, or $13\frac{1}{8}$, making $643\frac{1}{8}$.

NOTE.— When the unit figures are 1 and 9 the second figure of the product will be a cipher; thus: $61 \times 69 = 4209$.

By this simple rule, then, such numbers as 21×29, 22×28, 23×27; 31×39, 32×38, 35×35; 47×43, 48×42, 58×52; and up in the sixties, seventies, eighties, etc., may be multiplied together without effort. Also, 191×199, 192×198, 1992×1998, 1202×1208, 1307×1303, 798×792, 6993×6997, etc. (*See examples, page* 260)

EXAM. What will 198 hats cost at \$1.92 each?

In this, 2 and 8 make 10, and the 19's are alike. Say 2 times 8 are 16; set down in full, then, add 1 to 19 and say 20 times 19 are 380; this makes \$380,16, the cost.

$$\frac{198 \times 1.92}{\$380.16}$$

First: by casting out the 9's: cast the 9's out of both factors, as in the proof for Division, page 63, and reserve the excesses. Multiply these excesses together and from the result reject 9, also. Now, find the excess of 9's in the product, and if this be the same as found from the factors, the work is generally correct.

In the annexed example, the excesses of the factors are 4 and 3; their product is 12. Taking 9 from 12 leaves an excess of 3. Then, rejecting the 9's from the product, the excess is 3, also. (*See page* 63, *and note* 4, *page* 64.)

$$\begin{array}{lr} \left.\begin{array}{r}43276\ldots.4\\14682\ldots.3\end{array}\right\} & 12 \\ \hline 635378232 & 3\end{array}$$

Second: casting out 11's: commence at the units of the multiplicand and add all the digits in the odd places; then, all those in the even places, and from the former, increased if necessary, by 11, subtract the latter, and reserve the excess. Proceed in like manner with the digits of the multiplier, and reserve the excess. Multiply these two excesses and take the even from the odd (increased by 11, if necessary) and reserve the excess. Finally, in a similar manner, find the excess in the product, and if this be the same as that from the factors, the work is *generally* correct.

In the annexed example, the sum of the digits in the odd places of the multiplicand, is 12, and that in the even, 10; the excess is 2. Now, in the multiplier, the digits in the odd places make 9, and in the even, 12. Increasing 9 by 11 we get 20 from which 12 is taken, leaving an excess of 8. Then, 8 times 2 are 16, and taking the even from the odd in this, we say 1 from 6 leaves 5, the excess. Or, 11 from 16 leaves 5. Finally, the digits in the odd places of the product make 22, and those in the even, 17; the excess is 5, also, the same as found from the factors.

$$\begin{array}{lr} \left.\begin{array}{r}43276\ldots2\\14682\ldots8\end{array}\right\} & 16 \\ \hline 635378232\ldots.. & 5\end{array}$$

NOTE. — In all cases where the odds are less than the evens the odds must be increased by 11. Those figures can be applied to prove Addition, Subtraction and Division. The 11 is more reliable than 9, and may be used with advantage as a check figure to test the correctness of the footings of the Cash Book, Journal, etc. By applying both 9 and 11 to the same operation, an almost absolute certainty of its correctness would be obtained.

CANCELLATION.

Arithmetical calculations are frequently abbreviated in a wonderful manner by what is called *Cancellation*, that is, rejecting equal factors from numbers bearing to each other the relation of dividend and divisor; or, from multiplier and divisor in operations requiring multiplication and division.

Exam. 1. Multiply 476 by 312 and divide the product by 416.

$$
\begin{array}{r|l l}
\multicolumn{2}{c}{\underline{476 \times 312}} & \\
\multicolumn{2}{l}{1428..} & \\
\multicolumn{2}{l}{\quad 5712} & \\
\hline
1485 & 12 & \div \; \underline{416} \\
\hline
371 & 28 & \quad 104 \\
14 & 84 & \\
\hline
356 & 44 & \\
 & 60 & \\
\hline
357 & 04 & \\
 & 04 & \\
\end{array}
$$

357 is the required result, found by the methods already established.

Bringing cancellation to our aid, the same **result is** much more readily found as follows:

$$\begin{array}{ccc} 4 & & 3 \\ \cancel{416} & 476 & \cancel{312} \\ \cline{1-3} \cancel{104} & 1428 & \cancel{104} \\ \cline{2-2} & 357 & \end{array}$$

In this, we see at a glance that the multiplier, 312, is divisible by 3, giving 104; and the divisor, 416, is divisible by 4, giving 104 also. Now, multiplying 476 by 312 is the same as multiplying it by the factors, 3 and 104. Next, dividing by 416 is the same as dividing by the factors, 4 and 104. Multiplying 476 by 104, and dividing it at the same time by 104, will not change the number. Rejecting the common factors, 104, then, we simply multiply 476 by the remaining factor, 3, getting 1428, which is divided in turn by the remaining factor, 4, giving 357, as found by the long process.

EXAM. 2. Multiply 468 by 800 and divide the product **by 792.**

$$\begin{array}{rr|ll} \frac{\cancel{792}}{99} & 468 & .. & \frac{\cancel{800}}{100} \\ & 4 & 68 & \\ & & 4 & \\ \cline{2-3} & 472 & 72 & \end{array}$$

Dividing 792 and 800, in this, each by 8, we get 99 for divisor and 100 for multiplier. Annexing two ciphers (dots) to 468 multiplies by 100; then to divide by 99, according to the simplified method, we divide by 100 and add 1–100 as often as possible. Now, to divide by 100 we cut off the two ciphers (dots) which were annexed to multiply by 100. From this it will be readily seen that the position of the vertical line, in all such cases, is to the right of units, or through the decimal point.

EXAM. 3. Multiply 423 by 14 and divide the product by 728.

$$\frac{728}{104} \qquad \begin{array}{r|l} \multicolumn{2}{c}{423} \\ \hline 8 & 46 \\ & 32 \\ \cline{2-2} & 14 \end{array} \qquad \frac{14}{2}$$

Dividing 14 and 728 each by 7, in this, gives 2 for multiplier and 104 for divisor. Multiplying 423 then by 2 and dividing by 104 we get 8|14=8.134+. (See p. 17.)

EXAM. 4. Multiply 51 by 73 and divide by 49.

$$\frac{49}{98} \qquad \frac{51}{102} \qquad \begin{array}{r|l} 73 & .. \\ 1 & 46 \\ \hline 74 & 46 \\ 1 & 48 \\ & 2 \\ \hline 75 & 96 \end{array}$$

Doubling 49 and 51, in this, the multiplier is 102, and the divisor, 98. From what has been said in example 2, the position of the vertical line, here, is to the right of 73. Adding 2 times 73, now, placed in proper position, multiplies by 102. Then dividing by 98, we get 75|96=75.979+. (See pp. 19 and 21.)

On examining the process it will be seen that we have added 2-100 of 73, or 1|46; and in dividing by 98 we have added 2-100 of 74; then 2-100 of 1, or 1|48 and |02. Now, it is evident that the process can be abridged, thus:

$$\frac{49}{98} \qquad \frac{51}{102} \qquad \begin{array}{r|l} 73 & .. \\ 2 & 92 \\ & 4 \\ \hline 75 & 96 \end{array}$$

Adding 4-100 of 73, adds for 2 in 102, and also for 2 in the division by 98. Then 2 times 2, or 4, is added, which finishes the division by 98, and we have 75|96 as before.

EXAM. 5. Multiply 204 by 68 and divide the product by 214.

$$\frac{214}{107} \qquad \begin{array}{r|l} 68 & \\ 1 & 36 \\ \hline 69 & 36 \\ 4 & 83 \\ \hline 64 & 53 \\ & 35 \\ \hline & 88 \end{array} \qquad \frac{204}{102}$$

Dividing 214 and 204, each, by 2, gives 107 for divisor, and 102 for multiplier. The result is 64|88=64.82+.

In this it will be seen that 2-100, or 1|36, has been added; then 7-100, or 4|83, subtracted, which is evidently the same as subtracting 5-100 (the difference), so we simply subtract 5-100 of 68, or 3|40, as shown in the margin. This *adds* for the 2 in 102, and *subtracts* for 7 in 107 Then 7 times 4 (3 plus the 1 carried in subtracting 40), or 28, is added, to complete the division by 107. The result is 64|88, as before.

$$\begin{array}{r|l} 68 & \\ 3 & 40 \\ \hline 64 & 60 \\ & 28 \\ \hline & 88 \end{array}$$

EXAM. 6. Divide 748 times 416 by 412.

$$\frac{412}{103} \qquad \begin{array}{r|l} 748 & \\ 7 & 48 \\ & 21 \\ \hline 755 & 27 \end{array} \qquad \frac{416}{104}$$

Dividing 412 and 416 each by 4, the divisor is 103 and the multiplier 104. Now, we have to add for 4 in 104, and subtract for 3 in 103; their difference is 1, so we simply add 1; that is,

1-100 of 748, or 7|48. This completes the multiplication by 104, and subtracts for 3 in the divisor. But as the *addition exceeds* the subtraction, it is evident that 7, which is in reality a part of 748, immediately above it, has got to be multiplied by 3 (in 103), and the result, 21, subtracted. Taking 21 from 48 gives the remainder, 27, and adding 7 to 748 gives the quotient, 755.

EXAM. 7. What would be the result of taking $32.64, 416 times, and dividing the result into portions of $31.04 each?

$$\frac{3104}{\frac{776}{97}} \qquad \begin{array}{r|l} 416 & \\ 20 & 80 \\ & 60 \\ & 3 \\ \hline 437 & 43 \end{array} \qquad \frac{3264}{\frac{816}{102}}$$

Taking $31.04 and $32.64 as so many cents, throws off the decimals, and the multiplier becomes 3264, and the divisor, 3104. A moment's inspection, now, shows that each is divisible by 4, giving 776 and 816. These, in turn, are divided by 8, giving 97 and 102.

Then, adding 5-100 of 416, or 20|80, completes the multiplication by 102, and adds for 3, the complement of 97. Next, 3 times 20, and then 3 times 1 (carried in adding the remainders 60 and 80), set in proper position, completes the division for 97. Adding the several results, we get 437|43.

NOTE.— The value of the remainder, .43, is found by multiplying it by 4, and the product by 8 (the factors by which we divided in the example). The product is 1376, evidently cents in this case, or $13.76, which is not large enough to make a portion ($31.04). The answer, then, is, $31.04 could be taken 437 times, and $13.76 would remain.

EXAM. 8. A hatter exchanged 172 men's hats which cost him $3.35 each, for a number of boys' hats, at $1.68 each; how many of the latter did he receive?

REMARK.— In simplifying arithmetical operations, the *relation* which the numbers sustain to each other, must be carefully borne in mind. If, for instance, the numbers sustain to each other the relation of multiplicand and multiplier, whatever operation is performed on the one, to simplify the process, the *reverse* is performed on the other, that the product may not be changed; and if the relation be that of dividend and divisor, whatever operation is performed on the one, the *same* is performed on the other, that the quotient may not be changed.

To solve the above example, we multiply \$3.35 by 172, and divide the product by 1.68; thus:

```
   6          3          6
 168        335        172
----       ----       ----
1008       1005       1032
                      ----
                       344...
                         1720
                      -------
                      345|720
                        2|760
                      ---|---
                      342|960
                         | 24
                         |---
                         |984
```

Here, 335 and 172 bear to each other the relation of multiplicand and multiplier, and 168, that of divisor to each of the others, or, to their product.

To divide by 168 we multiply it by 6, getting 1008, a simple divisor; and multiplying 172 also by 6 we get 1032. Next, to multiply 335 by 1032, we take the former for multiplier and multiply it by 3 to get 1005, a simple multiplier. Multiplying 335 by 3, *divides* 1032 by 3, giving 344.

The terms now are 1008, 1005 and 344. Multiplying 344 by 1005, and dividing the product, 345720, by 1008, according to our established methods, we get 342|984=342.976.

Bringing Cancellation to our aid, we can still further simplify the foregoing process, thus:

$$\frac{168}{1008} \qquad\qquad \frac{335}{1005} \qquad\qquad \begin{array}{r|l} \multicolumn{1}{r}{172} & \\ \hline 344 & \ldots \\ 1 & 032 \\ \hline 342 & 968 \\ & 16 \\ \cline{2-2} & 984 \end{array}$$

It will be observed that, in the first process, 172 was multiplied by 6, and the product divided by 3, which, by cancellation, is simply multiplying by 2. Here, then, we simply multiply 172 by 2, getting 344. Next, adding 5 and subtracting 8 is subtracting 3, in this case 3–1000. So we subtract 3 times 344, placed in proper position, or 1|032. Now, since the *subtraction exceeds* the addition, it is evident that 1, or rather 2 (1 plus the 1 carried in subtracting), has yet to be multiplied by 8 (in 1008), and the product, 16, added to complete the division by 1008. The result is 342|984, as before.

EXAM. 9. How many pairs of men's shoes at $3.34 a pair can be got in exchange for 346 pairs of women's, worth $2.49½ a pair?

$$\frac{334}{1002} \qquad\qquad \frac{249\frac{1}{2}}{998} \qquad\qquad \begin{array}{r|l} \multicolumn{1}{r}{346} & \\ \hline \multicolumn{1}{r}{1038} & \\ \hline 259 & 5. \\ 1 & 037 \\ \hline 258 & 463 \end{array}$$

To solve this, $2.49½ is multiplied by 346, and the number of times $3.34 is contained in the product is the required number.

Multiplying 334 cents by 3 gives 1002 for a simple divisor, and

multiplying $249\frac{1}{2}$ cents by 4 gives 998 for a simple multiplier. Multiplying 334 by 3 multiplies 346 also by 3, giving 1038; and multiplying $249\frac{1}{2}$ by 4 *divides* 346 by 4; or, what amounts to the same thing, 1038 is divided by 4, giving 259.5. The terms, now, are 1002, 998 and 259.5. To multiply by 998 we use 1000 and subtract 2 (that is, 2 times the multiplicand), and to divide by 1002 we use 1000 also, and subtract for 2; in other words, we subtract 4-1000 of 259, or 1|036.

In multiplying by 998, however, it must be borne in mind that 259.5 has been multiplied by 2, while in dividing by 1002, only 259 has been multiplied by 2. In multiplying 259 by 4, therefore, 1 is added (found by multiplying the decimal .5 by 2, the complement of 998), making 1|037 to be subtracted; the result is 258|46 pairs.

EXAM. 10. How many acres of land at $\$32\frac{1}{3}$ an acre can be got in exchange for 176 acres, worth \$48.50 an acre?

$$\begin{array}{ccccc} 3 & & & & 2 \\ 32\frac{1}{3} & & 176 & & 48.50 \\ \hline 97 & & 88 & & 97 \\ & & \overline{264} & & \end{array}$$

Multiplying $32\frac{1}{3}$ by 3, gives 97 for divisor—multiplying 48.50 by 2, gives 97 for multiplier, also; and, being common factors, we reject both. Now, multiplying the *divisor*, $32\frac{1}{3}$ by 3, *multiplies* the *dividend*, 176, also, by 3; and multiplying 48.50, *one of the factors* of the multiplication, by 2, *divides* the *other factor*, 176, by 2. So we simply divide 176 by 2, and multiply the result, 88, by 3, to get 264 acres, the required number. Or, add 88 to 176, because multiplying by 3 and dividing by 2 is multiplying by $1\frac{1}{2}$.

NOTE.—Other examples, showing methods of abbreviation, might be added without limit, but those which are given will be sufficient by way of suggestion.

RULE OF THREE.

"*The Rule of Three*" (so called because there are always *three* numbers given to find a fourth) has reference to that part of Simple Proportion usually taught in arithmetic, *to find a fourth proportional to three given numbers.*

The resolution of this problem is the most important result of the theory of Proportion. On account of its great utility and its extensive application by merchants, accountants and others, it has been called the *Golden Rule.*

A thorough knowledge of this problem will enable the student to understand, with little effort, questions in Percentage, Interest, Discount, Commission, Profit and Loss, and other branches of arithmetic having the principles of Proportion for their basis.

Proportion is the equality of ratios.

Ratio is the relation which one number bears to another of the same kind, with regard to size or comparative value.

Thus, since 8 is double of 4, and 10 of 5, the ratio of 8 to 4 is equal to that of 10 to 5. The four numbers, 8, 4, 10 and 5 are proportionals and constitute a proportion, or analogy.

In this relation they are usually written thus:

As 8 : 4 :: 10 : 5; or, simply, 8 : 4 :: 10 : 5.

The first expression is read, *as* 8 *is to* 4 *so is* 10 *to* 5; and the second, 8 *is to* 4 *as* 10 *is to* 5, meaning that whatever relation 8 has to 4, the same relation exactly 10 bears to 5; in other words, whatever number of times 8 is greater or less than 4, 10 is as many times greater or less than 5.

Now, 8 : 4 is a ratio, meaning that 8 has a certain relation to 4; 10 : 5 is also a ratio, and both being equal, they might also be written thus:

$$8 : 4 = 10 : 5$$

and may be read as in the other cases, or, the ratio of 8 to 4 equals the ratio of 10 to 5.

The two numbers, 8 and 4, are the *Terms* of the first ratio, the first number is called the *Antecedent,* and the second, the *Consequent;* 10 and 5, are the *terms* of the second ratio, 10 being the *antecedent*, and 5 the *consequent.*

The *value* of a ratio is the quotient obtained by dividing the antecedent by the consequent; thus: the value of 8 : 4 is $\frac{8}{4}$, or 2; and of 10 : 5, $\frac{10}{5}$, or 2.

Since there are two terms in a ratio, and two ratios in a proportion, there must be at least four terms in every proportion.

The first and fourth terms of the proportion, or the *outside* numbers, 8 and 5, are called the *Extremes;* and the second and third terms, or the *middle* numbers, 4 and 10, the *Means.*

And when four numbers constitute a proportion *the product of the means will always be found to be equal to the product of the extremes.* Thus, in the proportion:

$$8 : 4 :: 10 : 5$$

4 multiplied by 10 equals 8 multiplied by 5.

Hence, *any three terms of a proportion being given, the fourth can be readily found.* Thus, if an extreme be wanting: *Multiply the two means together and divide the product by the given extreme;* the quotient will be the required extreme, and if a mean be wanting:

Multiply the extremes together and divide the product by the given mean; the quotient is the required mean.

EXAM. 1. What is the fourth term of the proportion

$$16 : 4 :: 24 : ?$$

Solution: $24 \times 4 = 96$; then, $96 \div 16 = 6$, the required term.
The proportion is now $16 : 4 :: 24 : 6$.
Proof: $24 \times 4 = 96$, the product of the means.
$16 \times 6 = 96$, the product of the extremes.

EXAM. 2. Complete the following proportion:

$$16 : ? :: 24 : 6$$

Solution: $16 \times 6 = 96$; then, $96 \div 24 = 4$, the second term.
In this, two extremes are given, and one mean, to find the other. The extremes are 16 and 6; they are multiplied together and the product, 96, divided by 24, the given mean; the quotient, 4, is the other mean.

From the foregoing principles and illustrations we derive the following:

RULE. (*General Rule.*) *To find a fourth proportional to three given numbers:*

I. *Arrange the three given numbers in a line, in succession, setting the one which is of the same kind as the required term, the third in order.*

II. *If, by the nature of the question, the required term is to be greater than the third term, put the greater of the other terms in the second place; but if the required term is to be less than the third, then put the less of the other two in the second place.*

III. *Multiply the second and third terms together and divide the product by the first; the quotient will be the required term.*

NOTES.— 1. If the first and second terms be not of the same denomination, they must be reduced to such.

2. If the third term be a compound number it must be reduced to its lowest unit.

EXAM. 3. If a person is to receive $840 for 9 months' services, how much ought he to receive for 90 days at the same rate?

$$9 \times 30 = \cancel{270} : \cancel{90} :: 840$$
$$3 : 1 \quad \overline{280}$$

The answer must be money, so we put $840 for the third term, and from the nature of the question the answer is to be less than $840; the fourth less than the third, therefore, the second must be less than the first. The second term, then, is 90 days and the first 9 months. Now, the first and second, although being of the same kind (time), are not of the same denomination; multiplying 9 by 30 gives 270 days, and the statement is: 270 days is to 90 days as $840 is to the required term. A glance at the two first terms now shows that each is divisible by 90, and consequently the ratio is as 3 : 1. So we simply divide 840 by 3, and the quotient, $280, is the answer.

$$\text{PROOF: } \left\{ \begin{array}{l} 840 \times 90 = 75600 \\ 280 \times 270 = 75600 \end{array} \right\} \text{ or, } \left\{ \begin{array}{l} 840 \times 1 = 840 \\ 280 \times 3 = 840 \end{array} \right\}$$

NOTE. — It may be well to remark that, in abbreviating operations in proportion by cancellation, the first and second, or the first and third terms (but *never the second and third*), are to be operated upon.

EXAM. 4. If a pole 6 feet 3 inches high casts a shadow $7\frac{1}{2}$ feet long, how high is a steeple whose shadow is 120 feet long?

$$\begin{array}{ccccc} \text{ft.} & & \text{ft.} & & \text{ft. in.} \\ 7\frac{1}{2} & : & 120 & :: & 6.3 \\ \cancel{15} & & 240 & & \cancel{75} \\ & & 1200 & & 5 \\ & & \multicolumn{3}{l}{100 \text{ feet.}} \end{array}$$

In this, height is required, therefore, 6 ft. 3 in. *height* is put in the third place; and from the nature of the question, the answer is to be more than this third term, consequently, the second term is greater than the first; 120 is, therefore, put in the second place.

Reducing the first and second terms, now, to the same denomination, halves, and the third term to its lowest unit, inches, we have: 15 : 240 :: 75.

Bringing cancellation to our aid, now, 15 and 75, the first and third terms, are each divided by 15, giving 1 and 5 (the 1 is omitted, as it does not affect the process, and the first term disappears).

Multiplying the second and third terms, next, that is, 240 by 5, gives 1200, which is of the same denomination as that to which the third has been reduced, namely, inches. Dividing 1200 by 12 (inches in a foot) we have 100 feet, the height of the steeple.

$$\textsc{Proof:}\quad \begin{array}{c} \left\{\begin{array}{r} 240 \times 75 = 18000 \\ 1200 \times 15 = 18000 \end{array}\right. \\ \text{or,} \qquad\qquad\qquad\qquad \text{or,} \\ \left\{\begin{array}{r} 120 \times 6\frac{1}{4} = 750 \\ 100 \times 7\frac{1}{2} = 750 \end{array}\right\} \begin{array}{r} 240 \times 5 = 1200 \\ 1200 \times 1 = 1200 \end{array} \end{array}$$

The product of the means equal to the product of the extremes.

EXAM. 5. If a contractor be paid \$64.35 for excavating 143 cubic yards, or loads of earth, how much ought he to receive for excavating 335 loads, at the same rate?

$$\begin{array}{rcrcr|l} 7 & & 3 & & & \\ \underline{143} & : & \underline{335} & :: & \underline{64} & \underline{35} \\ 1001 & & 1005 & & \underline{21} & \underline{45} \\ & & & & 150 & 15. \\ & & & & & \underline{600} \\ & & & & & 75 \end{array}$$

The answer to this must be more than \$64.35; hence, 335 is the second term of the proportion.

To multiply by 335, we multiply it by 3, to get 1005, a simple multiplier. Multiplying 335 by 3, *divides* 6435 by 3, giving 2145, to be divided by 143.

To divide by 143, it is multiplied by 7, giving 1001 for a simple divisor. Multiplying 143 by 7, *multiplies* 2145, also, by 7, giving 15015. The terms, now, are:

$$1001 : 1005 :: 150.15$$

Here, we are to multiply by 1005, and then divide by 1001. Now, to multiply 150.15 by 1000, we simply move the point three places to the right, getting 150150; and to divide this by 1000, we move the point three places to the left, getting for the result, 150.150, or, as shown in the example, 150|15., the dot representing the cipher. Hence, we simply draw the vertical line through the decimal point of \$64. 35, divide by 3, and multiply by 7, to get 150.15.

Annexing a cipher (dot) to 150|15, now, we simply add the 4-1000 (5—1), or .600; the required result is \$150.75 (see Cancellation, examples 5, 6, 7 and 8).

EXAM. 6. A builder is paid \$678 for 168 perches of masonry; how much ought he to receive for $337\frac{1}{3}$ perches, at the same rate?

$$\begin{array}{rcrcr|l}
6 & & 3 & & & \\
168 & : & 337\frac{1}{3} & :: & 678 & \\
\hline
1008 & & 1012 & & 1356 & ... \\
 & & & & 5 & 424 \\
 & & & & & 40 \\
\hline
 & & & & \$1361 & 384
\end{array}$$

In this, the answer is to be money, therefore, \$678 is put in the

third place; and, as more will be paid for 337⅓ perches than for 168, the larger of the two is put in the middle.

Multiplying 168 by 6, gives 1008 for a simple divisor; and multiplying 337⅓ by 3, gives 1012 for a simple multiplier. Now, multiplying 168 by 6, multiplies 678, also, by 6; and multiplying 337⅓ by 3, *divides* 678 by 3. But, multiplying 678 by 6, and then dividing by 3, is simply multiplying it by 2. Multiplying 678, then, by 2, gives 1356. The analogy, now, is: 1008 : 1012 :: 1356; and the product of the second and third terms divided by the first, is the answer.

To multiply 1356 by 1012, we annex three ciphers (dots), and to the result *add* 12 times 1356.

To divide by 1008, we cut off the three ciphers, and *subtract* 8 times 1356.

Now, adding 12, and subtracting 8, is adding 4; so we simply add 4 times 1356 (in this case 4-1000), or 5424, placed in proper position. The multiplication by 1012 is now completed, but the division by 1008 is not. We have still to subtract 8 times the partial quotient, 5, or rather, 8-1000, which is .040.

Subtracting 40, now, from 424, we get .384, or 38 cents; and adding 5 to 1356, we have $1361.38, the answer.

EXAM. 7. A drygoods dealer bought at auction 315 yards of silk for $352.80, and at the close of the auction he bought, at private sale, 214 yards more of the same material at the same rate; how much did he pay for the latter?

$$
\begin{array}{rcrcr|ll}
3 & & 2 & & & & \\
315 & : & 214 & :: & 352 & 80 & \\
\overline{105} & & \overline{107} & & \overline{117} & \overline{60} & \\
 & & & & \overline{235} & \cancel{20} & \\
 & & & & 4 & 70 & \\
 & & & & & \cancel{20} & \\
 & & & & \overline{239} & \overline{70} & .. \\
 & & & & & 2 & 10 \\
 & & & & & \overline{68} & \\
\end{array}
$$

The answer here is to be less than \$352.80, so 214 is put for the second term. Dividing 315 by 3 gives 105 for simple divisor, and 214 by 2 gives 107 for simple multiplier. Dividing 315 by 3 divides 352.80 by 3, also, giving 117.60; and dividing 214 by 2 *multiplies* 352.80, or rather, in this case, 117.60 by 2, giving 235.20.

The terms now are : 105 : 107 :: 235.20, and the product of the second and third divided by the first gives the answer. We have now to add the 7-100, and then subtract 5-100; this is adding 2-100. The 2-100 of 235 is 4|70. Next, we have to subtract 5-100 of the partial quotient, 4, to finish the division by 105; this is found to be .20.

It will be observed, now, that 20 is to be added to 70, and at the same time 20 is to be subtracted; so we simply set down 70 and add 235 and 4, getting 239|70.

To find the correct decimal, or cents, now, a vertical line is drawn to the right of 70, and two ciphers (dots) annexed. To this result 2-100 is to be added, and 5-100 subtracted, as in the other part of the process; this is the same as subtracting 3-100. The 3-100 of 70 is 2|10. Next, 5-100 of 2, or .10, is to be subtracted; or, simply, 2 altogether is subtracted and we get 68, the correct cents. The answer is \$239.68.

EXAM. 8. If 3168 bushels of wheat cost \$3200, what will 317 bushels cost at the same rate?

```
32..
3168 : 317|   :: 3200
----
  32     3|17
          | 3
      ----|--
       320|20
```

In this, 317 bushels will cost less than 3168 bushels, therefore, 317 is put in the second place. Then, we simply draw the vertical line to the right of 317, and setting that number under itself, two

places to the right; then 3, under the last result, two places to the right, and adding, we get $320.20, the required cost.

Reason: Taking 3200 for approximate divisor, in connection with 3168, we find the key to the division, 1, that is, 1-100, as has been explained in the article on Simplified Division. Multiplying 317, now, by 3200, and dividing the result by 3200 does not change 317. The divisor is not 3200, however, but 3168, so we must add the 32-3200, or rather its equal, 1-100 of 317; and this is done by simply setting 3|17, and then |03, in proper position, as seen in the example.

NOTE.— The process will be made clear by dividing 3168, the true divisor, and 3200, the third term of the analogy, by 32, using the factors 4 and 8. It will then be seen that we have simply multiplied 317 by 100, and divided the product, 31700, by 99, according to Rule III of Simplified Division.

EXAM. 9. If 1372 bushels of oats cost $617.40, what will 1470 bushels cost at the same rate?

```
14..
1372 : 1470 :: 617|40
  28    105       30|8700
                 ---------
                 648|27
                  12|96
                    |26
                 ---------
                 661|49|00
                       |98
```

1470 bushels will cost more than $617.40, therefore, 1470 is made the second term of the proportion.

To divide by 1372, we use 1400 as approximate in connection with it, and the key to the division is 2 (2-100). A glance at 1470 shows that it is divisible by 14, giving 105 for a simple multiplier. Dividing the second term, 1470, by 14, divides the first term, 1372, also, by 14, or rather the approximate, 1400, giving 100 for approximate divisor. Multiplying $617.40, now, by 105

(simply set 5 times that number two places to the right, and add), and dividing by 100, we get 648|27. Then, adding the 2-100 of the partial quotients, we get \$661.49. Continuing the division, we find the correct decimal, or cents, to be .50, .98 being equal to 1. The answer, then, is \$661.50.

NOTE.— If 1372, the real divisor, be divided by 14, instead of the approximate, 1400 (using the factors 2 and 7 to divide by), we find the new divisor is 98; and that we have simply multiplied \$617.40 by 105, and divided the product by 98.

Now, since the key to the division is found by dividing the difference of the divisors, or 28, by 14 (the significant part of 14..), the key thus found, in all such cases, *will be the difference between the approximate* (new) *divisor, found by cancellation, and the real* (new) *divisor.* Thus, 28 ÷ 14 = 2, the key; and 14.. ÷ 14 = 100, the *approximate* (new) divisor; then, 100 − 2 = 98, the *real* (new) divisor (all of which can be seen mentally).

EXAM. 10. If 2328 yards of silk cost \$2460, what will 310 yards cost at the same rate?

$$
\begin{array}{rr|l}
24.. & & \\
\underline{2328} : & 310 & :: \frac{2460}{102\frac{1}{2}} \\
72 & 17 & 05 \\
 & & 51 \\
\hline
 & \$327 & 56
\end{array}
$$

NOTE.— The real (new) divisor in this example is 97, found mentally, thus: 24.. ÷ 24 = 100, approximate (new) divisor; 72 ÷ 24 = 3; then, 100 − 3 = 97.

Dividing the first term of the proportion by 24, divides the third term, 2460, also, by 24, giving 102½ for multiplier. Multiplying 310, now, by 102½, and dividing by 97, we get \$327.56, the required cost. Setting 5½ times 310, or 1705, in proper position adds for 3 in the divisor, 97, and also for 2½ in the multiplier, 102½, and completes the multiplication. Then, 3 times 17, or 51, placed in proper position, is added to complete the division by 97. (*See remarks on Rule III, page* 38.)

COMPOUND PROPORTION.

Proportion is often of such a nature as to require a compound and a simple ratio, and is then called *Compound Proportion;* but a knowledge of simple proportion will enable us to solve compound with equal facility.

A compound ratio, as its name denotes, consists of two or more simple ratios, and can be reduced to a simple ratio by the following:

RULE. *To reduce a compound ratio to a simple one: Multiply all the antecedents together for a new antecedent, and all the consequents together for a new consequent.*

EXAM. 1. If it cost $3600 to supply 30 men with provisions for 24 days, the rations being 20 ounces per day, how much will it cost to supply 20 men for 36 days, the rations being 18 ounces per day?

In this example, it will be observed, there are three pairs of terms or couplets, namely, 30 men and 20 men, 24 days and 36 days, 20 ounces and 18 ounces; and there is a single term, $3600. Arranging the couplets, as shown in the margin, which we will call "noting the question"; setting the single term last, and under it, the interrogation point, to indicate that the corresponding term is wanting, we are prepared to set the antecedents and consequents of the respective ratios in their proper places.

Noting

men.	days.	oz.	
30 —	24 —	20 —	$3600
20 —	36 —	18 —	(?)

The answer to the question being money, $3600 is put for the third term in the following statement:

$$\begin{array}{rcl} 30 & : & 20 :: 3600 \\ 24 & : & 36 \\ 20 & : & 18 \\ \hline \cancel{14400} & : & 12960 :: \cancel{3600} \\ 4 & & \$3240 \quad\quad 1 \end{array}$$

To arrange the other terms in their proper places, now, we go back to the noting, and, taking the first couplet, men, in connection with the single or third term, we proceed by question and answer, thus: If it cost $3600 to supply 30 men, will it cost more or less to supply 20 men? The answer is evidently less; 20 is therefore put in the second place, and 30 in the first. Next, if it cost $3600 to get 24 days' supply, will it cost more or less to get 36 days' supply? More; therefore, 36 is the second term of the next ratio and 24 the first. Now, if it cost $3600 to supply 20 ounces of rations, will it cost more or less to supply 18 ounces? Less; and, therefore, 18 is the second term of the third ratio.

The compound ratio now consists of three simple ones, namely: 30 : 20, 24 : 36 and 20 : 18.

Multiplying the antecedents together, now, we get 14400 for a new antecedent, and the consequents together, and we get 12960 for a new consequent. The proportion, incomplete, now stands: 14400 : 12960 :: 3600; that is, three terms to find the fourth. Multiplying the second and third terms, and dividing the product by the first, as in simple proportion, will give the fourth proportional, or the answer.

Before proceeding with said multiplication and division, however, we see by inspection that the process can be abridged by Cancellation, 3600 being a measure, or exact divisor, for 14400. Dividing each of these numbers, then, by 3600, gives 1 for the third term and 4 for the first, and we simply divide the second term, 12960, by 4, getting $3240, the required fourth proportional, or the answer.

PROOF: $\left\{\begin{array}{l} 12960 \times 3600 = 46656000 \\ 14400 \times 3240 = 46656000 \end{array}\right\}$

or, $\left\{\begin{array}{r} 12960 \times 1 = 12960 \\ 3240 \times 4 = 12960 \end{array}\right\}$

The product of the means equal to the product of the extremes, taking either pair of ratios.

NOTE. — It is almost needless to remark, at this stage of the work, that, before multiplying the antecedents and consequents together, as in the example, recourse may be had to cancellation, dividing the first and second terms, or the first and third, by any numbers that will reduce or eliminate those terms.

PRACTICAL PROBLEMS.

Proportion may be greatly abridged by the use of aliquot parts.

An *Aliquot Part* of a quantity is such a part as, when taken a certain number of times, will exactly make that quantity. Thus, 4 is an aliquot part of 12, $2\frac{1}{2}$ of 10, $12\frac{1}{2}$ and $33\frac{1}{3}$ of 100, etc.

EXAM. 1. What is the cost of a cargo of iron weighing 259166 pounds, at \$36.16 per gross ton?

There are 2240 pounds in a gross ton, and by proportion, the statement is:

$$2240 : 259166 :: 36.16$$

Multiplying the second and third terms, and dividing by the first will give the solution, which, by the usual method, is quite tedious.

Now, by assuming as the price per ton, some measure, or

aliquot part, of 2240, as, for instance, 32, the foregoing problem can be abridged, thus:

$$\overset{7|0}{\cancel{2240}} : 25916|6 :: \overset{1}{\cancel{32}}$$

3702	371	=	price	@	$32 00
462	796	=	"	"	4 00
11	569	=	"	"	10
5	785	=	"	"	05
1	157	=	"	"	01
$4183	688	=	"	"	$36 16

If the price per gross ton were $32, the statement would be:

2240 : 259166 :: 32, and by cancellation, this becomes

70 : 259166 :: 1; 32 being contained in itself once, and 70 times in 2240. Multiplying by 1, now, and dividing by 70, gives the price of the given number of pounds at $32 per ton. To divide by 70, we cut off the cipher, and also one figure, 6, from the right of the dividend (this divides by 10), then dividing by 7, we get $3702.37, the price at $32 per ton. Next, we take aliquot parts of this for $4.16, the difference between the real and the assumed price, thus: $4 is $\frac{1}{8}$ of $32, therefore, the price at $4 will be the eighth part of that at $32, or, $462.79+ (the remaining decimals being mills, etc.). Then, 10 cts. is $\frac{1}{40}$ of $4, 5 cts. is $\frac{1}{2}$ of 10, and 1 c. is $\frac{1}{5}$ of 5. The sum of the several partial quotients, or, $4183.68, is the price at $36.16.

EXAM. 2. What is the cost of a cargo of iron weighing 276388 pounds at $23 per gross ton?

27638	8				
3948	40	=	price	@	$32
987	10	=	"	"	$8
123	3875	=	"	"	1
$2837	91	=	"	"	$23

Here, we simply divide the given number of pounds by 70, as in the preceding example, and we have the price at $32 per ton. The difference between this and the real price, $23, is $9, for which we take aliquot parts, thus: $8 is $\frac{1}{4}$ of 32, and $1 is $\frac{1}{8}$ of $8. Adding the price at $8 and $1, gives the price at $9, which, being taken from the price at $32, gives that at $23, or $2837.91.

NOTE. — The subtraction of the two numbers is performed at a single operation, as pointed out in short methods for Multiplication. (See note to example 14, page 78.)

EXAM. 3. What is the cost of 24840 pounds of coal at $3.50 per gross ton?

The statement of this problem by Proportion would be:

$$2240 : 24840 :: 3.50$$

and the product of the second and third terms, divided by the first, would be the answer.

We will here assume $28 as the price per gross ton, and we have:

$$\begin{array}{rl}
\overset{80}{\cancel{2240}} : \underline{2484} & \!\!|\,\underline{0} :: \overset{1}{\cancel{28}} \\
\underline{310} & \!\!|\,\underline{50} = \text{price @ } \underline{\$28} \\
\$38 & \!\!|\,81 = \text{“ “ } \quad 3.50
\end{array}$$

By cancellation, 2240 and 28 become 80 and 1; 28 being contained in itself once and 80 times in 2240. Dividing then by 80, as in the two preceding examples, we get $310.50, the price at $28 per ton. Now, $3.50 is $\frac{1}{8}$ of 28; therefore, the eighth part of the price at $28 is the price at $3.50. Dividing $310.50 by 8, we get $38.81, the required cost.

NOTE. — It may be well to remark, in connection with this method of abbreviation, that, although there are other aliquot parts, or measures, of 2240, the nnmbers 32 and 28 will be found the most convenient for practical

purposes. The price per gross ton will always decide which of the two is the more desirable to be taken as the assumed price.

FREIGHT.

When the price per gross ton is less than a dollar, we would proceed as in the following:

EXAM. 4. What is the freight on 23800 pounds of merchandise at 96 cents per gross ton?

$$\begin{array}{r|l|l} 23 & 80 & 0 \\ \hline 3 & 40 & = \text{price @ } 32 \text{ c.} \\ \hline \$10 & 20 & = \text{ “ \quad “ } \overline{96} \text{ c.} \end{array}$$

If the price per ton were $96, instead of 96 cents, in this example, we would cut off the right hand cipher, and divide by 7, as in the other examples, getting the price at $32. But cents are the hundredths of dollars, so we cut off two more figures from the right, and then proceed as in the other examples. In all such cases, then, *three* figures are cut off from the right, and the process will be as pointed out in the foregoing examples. $3.40 is the freight at 32 cents, and 3 times this, or $10.20, is the freight at 96 cents.

OATS.

What is the cost of 24160 pounds of oats at 58 cents per bushel, the bushel consisting of 32 pounds?

This problem, by Proportion, would be:

$$32 : 24160 :: 58$$

and the product of the second and third terms, divided by the first, would be the answer.

Assuming that the price per bushel is 32 cents, instead of 58, the statement would be:

32 : 24160 :: 32; and this by cancellation would be:
1 : 24160 :: 1. The price of the given number of pounds, then, at 32 cents per bushel is 24160 cents, or $241.60; that is, 1 cent per pound, and the process thus:

241	60	= price @	32c
483	20	= " "	64
30	20	= " "	4
15	10	= " "	2
437	90	= " "	58

At 1 cent per pound, or 32 cents per bushel, the price is $241.60. Doubling this gives the price at 64 cents per bushel, which is 6 cents per bushel too much. Then 4 cents is $\frac{1}{8}$ of 32; dividing $241.60 by 8 gives the price at 4 cents. Next, 2 cents is $\frac{1}{2}$ of 4; taking $\frac{1}{2}$ of $30.20, the price at 4 cents, gives $15.10, the price at 2 cents, and deducting both from $483.20, the price at 64 cents, gives $437.90, the price at 58 cents, or the answer. Or thus:

241	60	32
120	80	16
60	40	8
15	10	2
437	90	58

The process here is self-evident and requires no analysis.

NOTE. — The four last problems will suggest the method of abbreviating commercial calculations by the use of aliquot parts. Wheat, buckwheat, barley, etc., may be treated in like manner.

PERCENTAGE.

Per cent. from the Latin *per centum*, signifies *by the* 100. In business transactions, it means a certain part of every 100. Thus, 2 per cent. means 2 of every 100, and may signify 2 cents of every 100 cents, 2 dollars of every 100 dollars, 2 yards of every 100 yards, etc.

The character, %, is used in business transactions to represent the words *per cent.;* thus 2% means 2 per cent.

In Percentage, five quantities are concerned, namely:

The Base, *Rate per cent.*, *Percentage*, *Amount*, and *Difference.*

The Base is the number on which the percentage is computed.

The Rate per cent is the part of 100 taken.

The Percentage is the fourth proportional to 100, the rate per cent., and the base, taken in the order mentioned.

The Amount is the base plus the percentage.

The Difference is the base less the percentage.

Case I.

Given, the base and rate, to find the percentage.

Exam. 1. What is 4% of $350.

$$\begin{array}{r} 350 \times 4 \\ \hline \$14|00 \end{array}$$

To solve this, we multiply the base, $350, by 4, the rate per cent., and divide the product, 1400, by 100; simply cutting off two figures from the right; $14 is the percentage.

Reason: The foregoing is simply a question in proportion, expressed thus: If $4 be allowed on $100, how much ought to be allowed on $350, at the same rate?

And the analogy is: as $100, base, is to $350, base, so is $4, the percentage on $100, to the corresponding percentage on $350.

Thus:

$$100 : 350 :: 4$$

or, as $100 is to its percentage, 4, so is $350 to its percentage.

Thus:

$$100 : 4 :: 350$$

In either case, the product of the second and third terms divided by the first gives the fourth proportional. Hence,

RULE. *To find the percentage: Multiply the base by the rate per cent. and divide the product by* 100.

EXAM. 2. A.'s salary is $2500 a year; if he spend 10% for board, 6% for clothing, 5% for books, and 9% for other purposes, what are his yearly expenses?

NOTE.— When several rates refer to the same base, they may be added or subtracted, according to the nature of the question. Thus: 10% + 6% + 5% + 9% = 30%; then 30% of $2500 equals $750, his yearly expenses.

CASE II.

Given, the percentage and base, to find the rate.

EXAM. 1. What per cent of $350 is $14?

The question, fully expressed, is this: If $14 be allowed on $350, how much ought to be allowed on $100 at the same rate?

And the analogy is: As $350, base, is to $100, base, so is $14, the percentage on $350, to the corresponding percentage on $100.

Or, as the given base is to its percentage, so is $100, considered as a base, to its percentage. Thus:

$$350 : 14 :: 100$$

Multiplying the second and third terms, now, and dividing by the first, we get the rate on $100, or the rate per cent. Hence the

RULE. *Multiply the percentage by* 100 *and divide the product by the given base; the quotient is the rate.* Thus:

$$1400 \div 350 = 4\%$$

In the example, the percentage is 14 and the base 350. Annexing two ciphers to 14; that is, multiplying it by 100, the third term of the analogy, we have 1400; and dividing this by 350 gives 4, the rate per cent.

EXAM. 2. A merchant, failing, owes $14300, and his assets are only $10725; how much on the dollar can he pay to his creditors?

$$\begin{array}{r|l} 143 & 00 \\ \hline 1001 & \end{array} \qquad \begin{array}{r|l} 10725 & .. \\ \hline 75 & 075 \\ & 75 \\ \hline \end{array}$$

In this, $10725 is the percentage, and $14300 the base. Annexing two ciphers (dots) to the percentage, we have 1072500 to be divided by 14300. Cutting off the ciphers from both numbers and multiplying the remaining parts each by 7, we have 75075 to be divided by 1001.

The quotient is 75%; that is, 75 cents on the 100 cents, or the dollar.

CASE III.

Given, the percentage and the rate, to find the base.

EXAM. 1. \$14 is 4% of what number?

The question, fully expressed, is this: If \$4 be allowed on \$100, on what sum ought \$14 to be allowed? And the analogy is: As \$4, the percentage on \$100, is to its base, 100, so is \$14, the given percentage, to its corresponding base. Thus:

$$4 : 100 :: 14$$

Hence the

RULE. *Multiply the percentage by* 100 *and divide the product by the given rate; the quotient is the base.* Thus:

$$1400 \div 4 = \$350, \text{ the base.}$$

CASE IV.

Given, the amount and rate, to find the base.

EXAM. 1. What number increased by 4% of itself is equal to 364?

ANALYSIS: Since the amount is the base plus the percentage, it is evident that 104 is the amount of 100 (considered as a base). The question now is: If 104 be the amount of 100, of what number is 364 the amount?

And the analogy is: As 104, the amount of 100, is to its base, 100, so is 364, the given amount, to its corresponding base. Thus:

$$104 : 100 :: 364$$

Multiplying the second and third terms now, and dividing the product by the first, will give the base. Hence the

RULE. *Multiply the given amount by* 100 *and divide the product by* 100 *plus the rate; the quotient is the base.* **Thus:**

$$
\begin{array}{r|l}
364 & .. \div 104 \\
14 & 56 \\
\hline
349 & 44 \\
 & 60 \\
\hline
350 & 04 \\
 & 04 \\
 & \overline{}
\end{array}
$$

Annexing two ciphers (dots) to 364, we have 36400 to be divided by 104. The quotient is 350, the required base (Rule II, Division).

EXAM. 2. A merchant increased his stock in trade by 12% of itself, and then had \$3800; how much had he at first?

$$
\begin{array}{r|l}
\multicolumn{2}{r}{3800.. \div \;\; 112} \\
\hline
3420 & 000 \div 1008 \\
27 & 360 \\
\hline
\$3392 & 640 \\
 & 224 \\
\hline
 & 864
\end{array}
$$

In this, \$3800 is the amount, and 12 the rate. Annexing two ciphers (dots) to 3800, we have 380000 to be divided by 112. Multiplying both by 9, the divisor becomes 1008. The answer is \$3392.86.

CASE V.

Given, the difference and the rate, to find the base.

EXAM. 1. What number diminished by 4% of itself, is equal to 336?

ANALYSIS: Since the difference is the base less the percentage, it is evident that the difference of 100 (considered as a base) is 96. The question, now, is: If 100 be the base and 96 the difference, what ought to be the proportional base for the difference, 336?

And the analogy is: As the difference, 96, is to its corresponding base, 100, so is the given difference, 336, to its corresponding base. Thus:

$$96 : 100 :: 336$$

Multiplying the second and third terms now, and dividing the product by the first, will give the base. Hence the

RULE. *Multiply the given difference by* 100 *and divide the product by* 100 *less the rate; the quotient is the base.* Thus:

$$\begin{array}{r|l} 336 & .. \div 96 \\ 13 & 44 \\ & 52 \\ \hline 349 & 96 \end{array}$$

Annexing two ciphers (dots) to 336, we have 33600, to be divided by 96. The quotient is 349$\frac{96}{96}$, or rather 350, the remainder being 1.

APPLICATIONS OF PERCENTAGE.

The five rules of Percentage now established can be readily applied to problems in Commission, Stocks, Profit and Loss, Taxes, Insurance, etc. A few examples will suffice.

1. A merchant in Albany remits to his agent in Chicago $850.75 for the purchase of grain. The remittance includes commission at 2½%; how much will the agent expend for grain, and what will be his commission?

In this, $850.75 is the *amount*, and 2½ the *rate*. Now, what is the *base?*

To this, Case IV is applicable. Thus:

$$\begin{array}{r|l} 850 & 75 \div 102\frac{1}{2} \\ 21 & 25 \\ \hline 29 & 50 \\ & 525 \\ \hline \$830 & 025 \\ & 025 \\ \hline \end{array}$$

ANALYSIS: In this, we have to multiply first by 100, and then divide by 100 (in $102\frac{1}{2}$), consequently, \$850.75 undergoes no change and we simply draw the vertical line through the decimal point and make the proper correction for $2\frac{1}{2}$. To multiply 850 by $2\frac{1}{2}$, we conceive a cipher annexed and divide by 4 ($\frac{10}{4} = 2\frac{1}{2}$), or, $\frac{1}{40}$ of 850 as it stands, gives 21.25, which is placed in proper position and subtracted. Then, $\frac{1}{40}$ of 21, or .525, is placed in position and added, and finally, the $\frac{1}{40}$ of 1 (carried in adding |50 and |525), or |025, is subtracted. The result is \$830, the *base* of commission, which is the sum to be expended for grain. Then, $2\frac{1}{2}\%$ of \$830 = \$20.75, or \$850.75 — \$830 = \$20.75, the commission.

NOTE.— Commission is charged only on what is *expended* or *collected* by a person acting in the capacity of agent.

2. An agent sold real estate on commission at 3%, and returned to the owner, as net proceeds, \$2425; what was the price received for the property, and what was the commission?

In this, the net proceeds, \$2425, is the *difference* and 3 the *rate*. Now, what is the *base?*

Here, Case V is applicable. Thus:

$$\begin{array}{r|l} 2425 & .. \div 97 \\ 72 & 75 \\ 2 & 16 \\ & 6 \\ \hline \$2499 & 97 \end{array}$$

Annexing two ciphers (dots) to 2425, we have 242500, to be divided by 97. The quotient, by simplified division, is \$2500 (the remainder, .97, being 1), the required *base*, or what the property sold for; whence, by subtraction, we obtain the commission, \$75. Or, 3% of 2500 = \$75.

NOTE. — From a due consideration of the foregoing article, the student cannot fail to appreciate *the great advantage of a thorough knowledge of the principles of proportion* (given under the head of "The Rule of Three" in this work) *in all cases where percentage is concerned;* for, if a set rule should be forgotten, as frequently happens, a knowledge of proportion will enable us to recall it without difficulty.

If, for instance, we should forget the set rule for the last problem, we would reason thus: If, from property which sold for \$100, I receive \$97 as the net proceeds, from what sum ought I to receive \$2425 as net proceeds?

And the analogy is: 97 : 100 : : 2425, which gives the rule at once.

EXAMPLES FOR PRACTICE.

1. What is 12½% of \$5600? *Ans.* \$700.

2. What per cent. of \$720 is \$21.60? *Ans.* 3.

3. 18 is 25% of what number? *Ans.* 72.

4. What number increased by 15% of itself is equal to 644? *Ans.* 560.

5. What number diminished by 10% of itself is equal to 504? *Ans.* 560.

NOTE.— By solving those five examples by Proportion, the *reason* of the set rules will be impressed upon the memory.

INTEREST.

In computations in Interest there are five quantities to be considered, namely: the *Principal*, the *Interest*, the *Amount*, the *Rate* and the *Time*.

Principal has reference to money (or its equivalent) lent by one person to another, on condition that the borrower pays a certain sum to the lender for the use of the money.

Interest is the sum paid for the use of the principal, and is calculated on the basis of $100 as a standard principal, and one year as the time.

Amount is the principal and interest together.

Rate, or Rate per cent. per annum, is the sum allowed for the use of $100, for a year, per cent. meaning by the 100, and per annum, by the year.

The Time is that agreed upon by the parties to the transaction.

The most important problem in computations in interest is that in which the principal, the time and the rate are given, to find the interest or amount.

EXAM. 1. What is the interest of $376 for 2 months at 6 per cent?

ANALYSIS: This, in substance, is a question in compound proportion, and when fully expressed reads thus:

If $6 be paid for the use of $100 for 12 months, how much ought to be paid for the use of $376 for 2 months, at the same rate?

The statement, by proportion, would then be:

$$\begin{array}{r} 100 : 376 :: 6 \\ 12 : \quad 2 \quad\;\; \end{array}$$

A moment's glance at the terms of this proportion shows that the process can be abridged by cancellation; thus, dividing 12 and 6 each by 6 they become 2 and 1; next, dividing 2 thus found, and 2, the second term of the second ratio, each by 2, they disappear, and the proportion then is simply:

$$100 : 376 :: 1$$

Multiplying the second and third terms, in this proportion, and dividing the product by the first, we get $3.76, the fourth proportional, or the interest for 2 months.

From these facts we see that, *when the rate per cent is* 6, *and the time* 2 *months, or* 60 *days* (30 *days to a month*), *the interest of any principal will be just as many cents as there are dollars in the given principal.*

Taking this for our basis, it is evident that the interest for any number of months, at 6%, will be the interest for two months multiplied by *half* the given number of months, thus:

The interest of $376 for 10 months, at 6%, is $3.76 multiplied by 5 (half of 10 months), or $18.80; in other words, the interest for 2 months multiplied by 5 will be the interest for 10 months.

The interest of $376 for 3 years and 4 months, at 6%, is $3.76 multiplied by 20 (half of 40 months, the number in 3 years and 4 months), or $75.20.

And the interest of \$376 for 3 years, 4 months and 15 days, at 6%, is the interest for 3 years and 4 months plus $\frac{1}{4}$ of \$3.76 (the interest for 60 days), 15 days being $\frac{1}{4}$ of 60, thus:

\$3	76 =	the int. of \$376 for 2 mos., or 60 days, at 6%.			
75	20 =	"	"	3 years, 4 mos., or 40 mos.	
	94 =	"	"	15 days.	
\$76	14 =	"	"	3 years, 4 mos., 15 days.	

From this the interest at any other rate per cent. can be readily obtained by the method of aliquot parts; thus, if the rate were 7%, add $\frac{1}{6}$ of the interest at 6%, and if at 5%, deduct the $\frac{1}{6}$; if at 8%, add $\frac{1}{3}$ (2 being $\frac{1}{3}$ of 6), and if at 4%, deduct $\frac{1}{3}$; if 9%, add $\frac{1}{2}$, and if 3%, take the $\frac{1}{2}$; $7\frac{1}{2}$%, add $\frac{1}{4}$ ($1\frac{1}{2}$ being $\frac{1}{4}$ of 6), and if $4\frac{1}{2}$%, deduct the $\frac{1}{4}$, etc.

NOTE.—It need scarcely be remarked that, in computing interest, the business method is to reject the cents of the principal when less than 50, and when 50 or more, add \$1.

EXAM. 2. What is the interest of \$239.97 for 1 year, 5 months and 17 days @ 6%?

\$2	40 =	the interest for 2 mos., or 60 days.		
20	40 =	"	"	1 year, 5 mos., or 17 mos.
	60 =	"	"	15 days.
	08 =	"	"	2 days.
\$21	08 =	"	"	1 year, 5 mos., 17 days.

Here, we call the principal \$240; the interest on this for 2 months, or 60 days, is \$2.40. Multiplying this by $8\frac{1}{2}$ (half of 17 months, or 1 year and 5 months) gives \$20.40. For 17 days we take aliquot parts of 60, thus: 15 days = $\frac{1}{4}$ of 60; the $\frac{1}{4}$ of \$2.40 is 60 cents; then 2 days = $\frac{1}{30}$ of 60; the $\frac{1}{30}$ of \$2.40 is 8 cents,

or .08; and by addition we get $21.08, the interest for 1 year, 5 months and 17 days.

NOTE.—Should it be desirable to retain the cents of the principal, it will not, of course, affect the process, the figures to the right of the line being simply decimals, or cents and mills.

From the foregoing principles and examples we derive the following

EASY METHOD OF COMPUTING INTEREST ON ANY SUM, FOR ANY TIME, AT ANY RATE PER CENT., ON A BASIS OF 360 DAYS TO THE YEAR.

GENERAL RULE. (1.) *Draw a vertical line through the given principal, two places to the left of units; the result is the interest for* 2 *months, or* 60 *days, at* 6%: (2.) *multiply this by half the months in the given time (reducing the years, if any, to months):* (3.) *add, for days, such parts of* 60 *days' interest, as the days are aliquot parts of* 60; *the result will be the interest for the given time at* 6%, *the dollars being to the left, and the cents and mills to the right of the vertical line.*

Then, for any rate other than 6%, add or subtract the proper proportions, as pointed out in the foregoing analysis.

NOTE.—If the number of days be not an aliquot part of 60, say 25 days, for instance, then say 20 days = $\frac{1}{3}$ of 60, and 5 = $\frac{1}{4}$ of 20; the two results, when added, make 25. If 27 days, then 30 = $\frac{1}{2}$ of 60, and 3 = $\frac{1}{10}$ of 30; the difference is 27 (30 — 3), etc.

The foregoing is also the rule generally used for *Bank* or *Business Discount*, as it is called.

EXAM. 3. What is the bank discount on a note drawn at 3 months for $480.23 at 6%?

4|80 =	the interest for 2 mos., or 60 days.	
2|40 =	" " 1 mo., or 30 days.	
|24 =	" "	3 days (grace).
$7|44 =	" "	93 days

Deducting the interest (discount), $7.44, from $480.23, gives the present worth or the net proceeds.

EXAM. 4. What is the interest of $468 for 138 days at 6%?

12d = $\frac{1}{5}$ of 60
6d = $\frac{1}{2}$ of 12

```
    4|68
   -----
    9|36
     |936
     |468
  ------
  $10|76
```

```
13|8 ÷ 3|0
 4|18
```

Dividing 138 days by 30, as shown in the margin, we get 4 months and 18 days, the process is then according to the rule.

Now, since the interest of any sum of money for 60 days, or 2 months, at 6%, is as many cents as there are dollars; that is 1% of the principal, the interest for 6 days is a tenth of that for 60 days, or 2 months, and for 1 day, a sixth of that for 6 days; for 600 days, or 20 months, the interest is ten times that for 60 days, or 2 months and for 6000 days, or 200 months it is ten times that for 600 days, or 20 months; in other words, any sum of money will double itself, or the interest will equal the given principal, in 6000 days, 200 months, or $16\frac{2}{3}$ years, at 6%, simple interest, thus:

The interest of $5480 for 6000 days or 200 months = $5480.
" " $5480 for 600 days or 20 months = $548.0
" " $5480 for 60 days or 2 months = $54.80
" " $5480 for 6 days .... = $5.480
" " $5480 for 1 day................ = .913

And from this the interest for any time can be readily found, as illustrated in the following:

EXAM. What is the interest of $5498.70, for 3 months and 10 days, at 6 per cent.?

In this, counting 30 days to the month, the time is 100 days, and we simply cut off one figure from the dollars to get the interest for 600 days; then a sixth of this, or $91.645, is the interest for 100 days, or 3 mo., 10 da.

$549	8.70	=600da.
$91	645	=100 "

EXAM. What is the interest on a note for $7560, dated Jan. 31, 1906, and payable May 5th, following, at 6 per cent.?

Counting Feb. 28 days, Mar. 31, April 30 and May 5, the time is 94 days; and the interest for 100 days is $126 from which $7.56, the interest for 6 days is deducted; this leaves $118.44, the interest for 94 days.

$756	0	=600da.
126		=100 "
7	56	= 6 "
$118	44	= 94 "

Or,

The interest for 60 days=$75.60; for 30 days $37.80; for 3 da. a tenth of this, or $3.78 and for 1 da. a third of this, or $1.26, making, when added, the required interest.

$75	50	=60 da.
37	80	=30 "
3	78	= 3 "
1	26	= 1 "
$118	44	=94 da

EXAM. What is the interest of $5840, for one year, 8 months and 15 days, at 7 per cent.?

In this, 1 yr., 8 mo. = 20 mo. and at 6%, the interest is $584.

Then, the interest for 15 da. at 6% is one fourth of 60 days' interest, or of $58.40; this gives $14.60 which is added, making $598.60 for the given time at 6% To this we add the interest at 1%, or one-sixth of that at 6%, $99.766, which gives $698.366, the interest at 7%.

$584	0	=20 mo.
14	60	=15 da.
$598	60	=6%
99	766	=1%
$698	366	=7%

NOTE.—If the rate were 5%, $99.766, or one-sixth of 6% would be deducted; if 4%, a third of 6% or 2% would be deducted; if 7½% a quarter of that at 6% would be added 1½ being a quarter of 6, etc. (*See page* 124.)

RULE. *To compute interest at any rate per cent. for any number of days, on the basis of* 360 *days: Multiply the principal by double the rate; multiply the result by the number of days; cut off five figures from the right, counting from the decimal point always, and add a third and a sixth of that third.*

EXAM. What is the interest of $50000 for 14 days, at $3\frac{1}{2}$ per cent.?

In this, multiplying by 7 (double $3\frac{1}{2}$) and then by 14, is the same as multiplying by 98 (7×14) at once. Multiplying 98 by 5, and annexing the ciphers, we get $4900000. Cutting off five places, and adding a third and a sixth of that third, we get the required interest, to five places of decimals.

$50000 × 98	
49	00000
16	33333
2	72222
$68	05555

And to compute interest at any rate, for any number of days, on the basis of 365 *days, in other words, exact interest:*

RULE. *Multiply the principal by double the rate; multiply the result by the number of days; cut off five figures from the right, counting from the decimal point always, and add a third, a tenth of that third and a tenth of that tenth.*

EXAM. What is the interest of $256800 for 1 day at 2 per cent. (365 da.)?

Here, the time being only 1 day, we simply multiply by 4 (double the rate); then, cutting off five places, and adding a third, a tenth of that third and a tenth of that tenth, we get the interest, correct to five places of decimals.

$256800 × 4	
10	27200
3	42400
	34240
	3424
$14	07264

NOTE. — This rule will be found useful in finding the interest on *daily balances.* (*See page* 220.)

Both the foregoing rules will be found more fully explained in the Appendix (pp. 177 and 178) together with the reason of the rules.

PARTIAL PAYMENTS OR INDORSEMENTS.

The United States Courts have decided that,

I. "The rule for casting interest when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due.

II. "If the payment exceeds the interest the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due.

III. "If the payment be less than the interest the sur plus of interest must not be taken to augment the principal, but the interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal, and the interest is to be computed on the balance as aforesaid." — *Decision of Chancellor Kent.*

EXAM.

$1000. ALBANY, N. Y., *May* 1, 1885.

Two years after date I promise to pay to A. B., or order, one thousand dollars, with interest, value received.

C. D.

On this note were indorsed the following payments:

Jan. 1, 1886,	received	$150.
Sept. 1, 1887,	received	75.
Jan. 1, 1888,	received	200.

What remained due May 1, 1888?

PROCESS:

Principal on interest May 1, 1885..................		$1000	
Interest from May 1, 1885, to Jan. 1, 1886 (8 mos.)@6 %,		40	
Amount...........		$1040	
First payment, Jan. 1, 1886......................		150	
New principal......		$890	
Interest from first payment to Sept. 1, 1887 (20 mos.)..		89	
Second payment, Sept. 1, 1887 (less than interest).	$75		
Interest on $890 from Sept. 1, 1887, to Jan 1, 1888 (4 mos.)		17	80
Amount..........		$996	80
Third payment January 1, 1888	$200		
Sum of second and third payments		275	
New principal......		$721	80
Interest from Jan. 1, 1888, to May 1, 1888 (4 mos.)...		14	44
Balance due May 1, 1888		$736	24

Hence the

RULE I. *Compute the interest on the given principal to the time of the first payment, and, if less than the payment, add it to the principal and subtract the payment from the amount; the difference will be the new principal.*

II. *But if the payment be less than the interest, let the account stand* (noting the payment on the document) *till the next payment, when, if the sum of the payments shall equal or exceed the interest then due, add the interest to the new principal and subtract the sum of the payments from the amount; the difference will be the new principal, with which proceed as before.*

NOTE.— This rule applies to bonds, mortgages and other obligations bearing interest.

BANK DISCOUNT.

Bank Discount is an allowance made to a bank for the payment of a note before it becomes due.

The money received for the note when discounted is the *Proceeds*, and is equal to the face of the note less the discount.

The *Face* of a note is the sum made payable by the note.

Three days, called *Days of Grace*, are allowed on a note, after the time it is nominally due, before it is legally due.

Thus, a note drawn on April 8, at 2 months, would not be legally due till June 11.

The person by whom a note is signed is the *Maker* or *Drawer* of the note.

The person in whose favor, or to whose order the note is payable, is called the *Payee;* and the *Holder* is the owner of the note.

The *Maturity* of a note is the expiration of the days of grace.

To *Indorse* a note is to have the payee or holder write his name across the back of it.

NOTE. — An indorsement makes the indorser liable for the payment of a note if the maker fails to pay it when due.

The following is the usual custom of borrowing money at banks: The borrower presents a note, either made or indorsed by himself, payable at a future specified time. Interest is calculated on the face of the note *for the time it has to run from the date of discount;* this interest is deducted from the face and withheld by the bank in consideration of advancing the money before the note matures. Hence the

RULE. *To find the discount and the proceeds of a note:*

I. Find the interest on the face of the note for three days more than the time specified; *this interest is the discount.*

II. Subtract the discount from the face of the note; *the difference is the proceeds.*

NOTE. — When a note is given in settlement of an account, the business method is to *add the interest to the debt* and draw the note for the full amount. But sometimes notes are drawn promising to pay "*with interest*" (not including the interest with the debt). In such cases the *amount*, that is, the debt and interest together, is the face of the note, or sum made payable, and must be made the basis of discount.

EXAM.

$1260.15 ALBANY, N. Y., *Jan.* 17, 1889.

Ninety days after date I promise to pay to the order of Case & King, one thousand two hundred and sixty and $\frac{15}{100}$ dollars, at the Exchange Bank, for value received.

JOHN JONES.

Suppose Case & King discounted the foregoing note on February 4, what would be the discount, and what the proceeds?

ANALYSIS: In the first place we find the day of maturity, thus: Counting 90 days after January 17, we find the nominal day to be April 17, thus: subtracting 17 from 31 leaves 14 days for January, then February 28 days, March 31 days, and 17 days of April make

90. To this we add 3 days grace, and the day of maturity is April 20. Interest is computed on the note, now, from the date of discount, February 4 to April 20, at 6%. The number of days from February 4 to April 20 is 75, and the interest of $1260.15 for 75 days at 6% is found thus:

$$\begin{array}{r|l|l|l}
12 & 60 = & \text{interest for} & 60 \text{ days.} \\
3 & 15 = & \text{"} & 15 \text{ "} \\
\hline
\$15 & 75 & & 75 \text{ "}
\end{array}$$

The discount, then, is $15.75, and the proceeds $1260.15 minus $15.75, or $1244.40.

NOTE. — For business method of computing interest, see note, page 124; also general rule for interest, page 125.

Although the foregoing rule is that which is employed in actual practice, it is founded on a principle radically false, and always *gives the discount too large, and consequently the present worth too small, by the interest of the true discount.*

The *true* present worth of any debt is such a sum as would, if lent at interest at the assigned rate, amount to that debt at the time at which it would have been due. Hence the

RULE. *To find the true present worth of a debt:* As $100 plus its interest for the given time, and at the given rate per cent., is to $100, so is the debt to its present worth.

Thus, the true present worth of the note in the last example is found as follows:

The interest of $100 for 75 days at 6% is $1.25; adding this to $100, and proceeding according to the rule, we have the following analogy: $101.25 : $100 :: $1260.15 : $1244.59, the present worth.

Subtracting \$1244.59 from \$1260.15, the face of the note, gives \$15.56, the *true* discount, or 19 cents less than was found by the bank method. And the interest of \$15.56, the true discount, for 75 days at 6%, is found to be 19 cents.

The *reason of this rule:* \$101.25 is the *amount* of \$100 for the given time and rate, and \$1260.15 is the *amount* of \$1244.59 for the same time and rate; that is, *the amount of* \$100 *is to the principal which produced that amount as any other given amount is to the principal corresponding to the said amount.*

It is evident, also, that we might use the amount of \$1, and \$1 itself, or the amount of *any sum whatever*, and that sum itself, for the two first terms of the analogy; but it is generally more easy to use \$100 and its amount.

RULE. *To find the face of a note, the proceeds being given:* As the proceeds of \$100 is to \$100, so is the proceeds of the note to its face.

EXAM. 1. For what sum must a note be drawn at 4 months to net \$750 when discounted at 6%?

NOTE.—The days of grace having been abolished in the State of New York, are not taken into account in the following examples.

ANALYSIS: The interest of \$100 for 4 mo. at 6% = \$2. Deducting this from \$100 gives \$98, the proceeds of \$100.

Then, As $\$98 : \$100 : : \$750 : x = \frac{100 \times 750}{98} = \$765.30$

the sum for which the note must be drawn:
that is, the proceeds of a note whose face is \$100, *is to that face, as the proceeds of any other note is to its face.*

The division by 98 is performed by the simplified process: Dividing first by 100, 2 times 750 = 1500, and 2 times 15 = 30; are set in proper positition and added; the sum is \$765.30

750	00
15	00
	30
\$765	30

(*For a more simple method to find the face of a note, see p.* 214.)

RULE.—*To find the interest corresponding to a given rate of bank discount:*

I. Assume \$100 as the amount or face of the note, and find the discount and the proceeds of that amount for the time the note has to run. Then,

II. Apply the following proportion:

$$\left.\begin{array}{lcl}\text{As the given time} & : & 1\text{ year}\\ \text{the principal} & : & \$100\end{array}\right\} : : \text{the interest} : \text{the rate.}$$

EXAM. What rate of interest is paid, when a note payable in 60 days is discounted at 2% a month?

ANALYSIS: 2% per mo. = 24% per year.
The discount of \$100 for 60 da., or 2 mo., at 24% = \$4.
Deducting this from \$100 gives \$96, the proceeds.

Here, then, having assumed \$100 as the amount; \$96 is the principal, \$4 the interest and 2 mo. the given time, and we have the following proportion:

$$\left.\begin{array}{lcl}\text{As } 2\text{ mo.} & : & 12\text{ mo.}\\ \$96 & : & \$100\end{array}\right\} : : \$4 : \text{the rate.}$$

Reducing this to a simple proportion, we have: 192 : 1200 : : 4
Multiplying the second and third terms, now, and dividing by the first, we have: $\frac{1200 \times 4}{192} = 4800 \div 192 = 25\%$, the rate.

Or, having found the discount and the proceeds of \$100; draw a vertical line and set the given time and the proceeds on the left; and on the right set 1 year, \$100 and the discount. Divide the product of the numbers on the right by the product of those on the left, and the quotient is the rate.

$$\begin{array}{r|l} 2\text{ mo.} & 12\text{ mo} \\ \$96 & \$100 \\ & \$4 \\ \hline 192 & 4800\ (25\% \end{array}$$

NOTE.—The reader need scarcely be told that the numbers on both sides may be cancelled, if desirable; in this case they can be cut down to $100 \div 4 = 25\%$.

RULE.—*To find the rate of bank discount corresponding to a given rate of interest:*

Assume \$100 as the proceeds of the note and find the interest and the amount of that for the time the note has to run. Then,

$$\left.\begin{array}{ll}\text{As the given time} & : \text{1 year} \\ \text{the principal} & : \$100\end{array}\right\} : : \text{the interest} : \text{the rate.}$$

EXAM. A broker buys 30 day notes at such a discount that his money earns him $2\frac{1}{2}\%$ a month; what is the rate of discount?

ANALYSIS: $2\frac{1}{2}\%$ per mo. = 30% per year.
The interest of \$100 for 30 da. or 1 mo. at 30% = \$2.50.
Adding this to \$100 gives \$102.50, the amount.

Here, then, \$102.50 is the principal, \$2.50 the interest and 1 mo. the given time, and we have the following proportion:

$$\left.\begin{array}{ll}\text{As 1 mo.} & : \text{12 mo.} \\ \$102.50 & : \$100\end{array}\right\} : : \$2\ 50 : \text{the rate.}$$

and this now becomes: $\$102.50 : 1200 : : \$2.50 = \frac{1200 \times 2.50}{102.50} = 300000 \div 10250 = 29\frac{11}{41}\%$, the rate.

Or, having found the interest and the amount of \$100; draw a vertical line and set the given time and the amount on the left; next, set 1 year, \$100 and the interest, on the right. Divide the product of the numbers on the right by the product of those on the left; the quotient is the rate.

$$\begin{array}{r|l} \text{1 mo.} & \text{12 mo.} \\ \$102.50 & \$100 \\ & \$2.50 \\ \hline 10250 & 300000\,(29\frac{11}{41}\% \end{array}$$

NOTE.—It may be well to remark that the proportion given in connection with the two foregoing rules, is applicable to those problems in Interest where the principal, the interest and the time are given to find the rate.

AVERAGING ACCOUNTS.

The rule which determines the just time to pay, in *one* payment, several debts due at different times, is called *Equation of Payments*, or *Average.*

Exam. 1. On January 1, 1888, Jno. Dwyer bought of Wm. Prior

A bill of drygoods amounting to	$300 @ 2 months,	
A bill of hats amounting to	400 @ 3 months,	
And sundries amounting to	500 @ 4 months,	

at what time may the whole be paid without loss to either party?

Note.—The time which elapses before a payment is due is called the *Term of Credit;* and each item of a book account draws interest from the time it is due, which may be either at the date of purchase, or after a specified time of credit.

To find the average date of the foregoing account, we will assume that each item was due at the date of purchase; in other words, that they were cash transactions.

On this assumption, it is evident that the purchaser would owe the merchant, at the end of each term of credit, not only the

amount of each bill of goods, but also the interest on each amount for the time, thus:

Amount of account	$300,	int. for 2 mos. @ 6%	=	$3.00
" "	400,	" 3 " "	=	6.00
" "	500,	" 4 " "	=	10.00
	$1200			$19.00

The transactions were *not* for cash, however, but on *time*. The purchaser, therefore, is entitled to the $19 interest; in other words, *he is entitled to hold the* $1200 *for such time after January* 1, *as it would take that sum to give* $19 *interest at* 6% *per annum.* The question, now, is, in what time will $1200 principal give $19 interest, if $100 principal give $6 in 12 months? The proportion, or analogy, for this problem, would be:

$$\begin{array}{c} \$1200 : \$100 :: 12 \text{ mos.} \\ 6 : \quad 19 \end{array}$$

and this, by cancellation, becomes:

$$6 : 19 :: 1$$

Then, multiplying the second and third terms, and dividing by the first, we have $19 \div 6 = 3\frac{1}{6}$ months, or 3 months and 5 days.

If, then, the purchaser paid the entire $1200 of account, 3 months and 5 days after January 1, there would be no injustice on either side.

Counting 3 months and 5 days from January 1, we find the average date to be April 6th.

EXAM. 2. On January 1, 1889, a man gave 3 notes, the first for $500 payable in 30 days; the second for $400 payable in 60 days; the third for $600 payable in 90 days. What is the average term of credit, and what the equated time of payment?

PROCESS.

Interest on	$500	for	30 days	@ 6% =	$2 50
"	400	"	60	" =	4 00
"	600	"	90	" =	9 00
	$1500				$15 50

Here, the interest of the several payments for the respective terms of credit is $15.50, and the sum of the payments, $1500. Now, in what time will $1500 give $15.50 interest, if $100 give $6 in 12 months? The following is the analogy:

$$\$1500 : \$100 :: \cancel{12}\ \text{mos.}$$
$$\cancel{6} : \quad 15.50 \quad 2$$

And, by cancellation, 6 disappears and 12 becomes 2. Then we have $15.50 multiplied by 2, or $31, which is multiplied in turn by 100, giving $3100; dividing this last, now, by $1500, gives $2\frac{1}{15}$ months, or 62 days, the average term of credit; and the equated time of payment, March 4. Hence the following

RULE. *Find the interest on each payment for its term of credit at* 6 *per cent., and add the results; multiply double the interest thus found by* 100, *and divide the product by the sum of the payments; the quotient will be the average term of credit in months.*

NOTE.— If the sum of the payments be not contained in the product as found above, multiply said product by 30, then divide, and the quotient will be the average term in days.

The foregoing is called a Simple Equation, or Simple Average, having reference only to one side of the account; and the terms of credit begin at the same date.

When both debits and credits, or both sides of an account, are to be considered, and the terms of credit begin at different dates, the process is called a Compound Equation, or Compound Average; as illustrated in the following Ledger account:

Dr. RICHARD ROE. **Cr.**

1888.					1888.				
Aug.	3	To Mdse	240	00	Aug.	12	By Cash	500	00
"	10	" "	350	00	Sept.	30	" "	300	00
Sept.	1	" "	280	00					
Oct.	15	" "	230	00					
Nov.	26	" "	158	00					

Opening the Ledger, we find Richard Roe's account as above; what is the average date?

PROCESS.

		Time.	Amounts.		Interest.				Time.	Amounts.		Interest.	
Aug. 3	to Nov. 26	115 days	240	00	4	60	Aug. 12	to Nov. 26	106 days	500	00	8	83
" 10	"	108 "	350	00	6	30	Sept. 30		57 "	300	00	2	85
Sept. 1	"	86 "	280	00	4	01							
Oct. 15	"	42 "	230	00	1	61							
Nov. 26	"	0	158	00	0	00							
			1258	00	16	52				800	00	11	68
			800	00	11	68							
			458	00	4	84							

First, assuming that each item was due at the date of purchase, and that Mr. Roe wished to settle his account on November 26 (the latest date of the account), it is evident that each item bears interest from its respective date to the date of settlement, and that on November 26 he owed $1258 of account and $16.52 interest, as shown in the table opposite. Next, we find that he is credited with $500 and $300; he is now entitled to interest on each item from its respective date to November 26, the date of settlement, as shown in the table, viz.: $11.68. He owed the difference between both sides of the account; that is, $458 of account and $4.84 interest. He has, therefore, had the advantage of such time as it would take $458 to give $4.84 interest. The question now is, when should the account have been settled so that no interest would accrue? At such time before November 26 as it would take $458 to accumulate $4.84 interest at date of settlement. Multiplying double the interest, $4.84, or $9.68, now, by 100, we get $968, which is divided by $458, the balance of account, to get 2 months and 3 days, or 63 days. Counting 63 days *back* from November 26, we find the average date to be September 23.

NOTES.—1. If the balance of interest were on the credit side of the account, we should count *forward* from November 26th, as it is evident Mr. Roe would, in that case, be entitled to the time.

2. Had there been a term of credit on the above account, say 2 months, for instance, then, the required date would be September 23d, plus 2 months, or November 23d,

3. If the terms of credit differed; that is, suppose each item in the above account was on 30 days' time, except that of September 1, on which there is a term of, say, 3 months. In that case we would not take November 26th as the date of settlement, but the date on which the item having the longest term of credit falls due, which would now be December 1. On the other hand, allowing the dates to be as in the account, except that on September 30th, Mr. Roe gives his note for, say, 2 months, instead of cash; then the maturity of said note, that is, December 3d, would be the date to which the interest on the account would be computed. The date to which interest is computed, in such cases, is called the *focal* date.

4. There may be such a combination of debits and credits, that the average date will be earlier or later than any date of the account.

5. The rate is optional, in computing interest on the account, but the same rate must be used for both sides of the account. The 6 per cent rule, and 60 days basis, is, perhaps, preferable to any other.

Hence the

RULE. (1.) *Find the interest of each item on the debit and credit sides of the account, at* 6 *per cent., from each respective date to the latest date on the account, whether such date be on the debit or credit side; the difference will be the balance of interest on the account:* (2.) *multiply double the balance of interest by* 100, *and divide the product by the balance of account; the quotient will be the time to be counted Backward from the focal date if the balance of interest and the balance of account be on the same side; but Forward if the balances be on opposite sides.* (See note 3 to foregoing analysis.)

NOTES.— 1. It is scarcely necessary to remark, that, in averaging an account, we use the business method in computing the interest, namely: reject the cents in each item when less than 50, and when 50 cents, or more, add a dollar. The balance found from the items thus used will be the divisor in such cases.

2. In getting the interest for days, we take 60 days' interest as the basis; thus, to find the interest of \$240 for 115 days (the first item on the Dr. side of the table), \$2.40 is 60 days' interest; then 2 times \$2.40, or \$4.80 is 120 days' interest. From this we subtract 5 days' interest or $\frac{1}{12}$ of \$2.40, which is 20 cents, giving \$4.60, the interest for 115 days, and so with the remaining items.

3. If the number of days be large, divide by 60; multiply the interest for 60 days by the quotient, and add, for the remainder, the proper proportions of 60 days' interest. Thus, what is the interest of \$240 for 243 days at 6 per cent?

$$\frac{24|3 \div 6|0}{4}$$

Dividing 243 days by 60, gives 4 for quotient, and 3 for remainder. Multiplying \$2.40 (60 days' interest) by 4, gives \$9.60, or 240 days' interest; then, 3 days equal $\frac{1}{20}$ of 60 days, or 12 cents, making \$9.72, the interest for 243 days. Again, the interest of \$240 for 316 days, for instance, would be 5 times 60, plus 16 days (31|6 ÷ 6|0 = 5..16). For 16 days we would say 10 = $\frac{1}{6}$ of 60; and 6 = $\frac{1}{10}$; the sum of the results is 16 days' interest, etc.

PROFIT AND LOSS.

Profit and *Loss* are commercial terms, having reference to the gain made, or the loss sustained, in the course of business.

Gains and losses are usually estimated at some rate per cent. on the money first expended or invested.

NOTE.—It should be particularly remarked, that, *by the gain or loss per cent. is to be understood the sum that would be gained or lost at the given prices*, not on a hundred dollars' worth sold, but *on a hundred dollars laid out in first cost, and in charges, if there be any.*

RULE I. *The first cost and the selling price being given to find the gain or loss per cent.:* As the first cost is to the gain or loss on that cost, so is $100 to the gain or loss per cent.

EXAM. 1. If tea be bought at 40 cents and sold at 50 cents per pound, what is the gain per cent.

$$40 : 10 :: 100$$

$$\underline{10}$$

$$4|0)100|0$$

$$25\%$$

Here, 50 — 40 = 10 cents, the gain on 40 cents; then, as 40 cents (the first cost) : 10 cents (the gain on 40 cents) :: $100 (regarded as first cost) : 25, the gain on $100.

RULE II. *To find how a commodity must be sold to gain or lose a certain rate per cent.:* As \$100 is to the gain or loss on \$100, or per cent., so is the first cost to the gain or loss on that cost; and from this and the first cost, the selling price will be found by addition or subtraction.

EXAM. 2. How must tea which cost 60 cents per pound be sold to gain 20%?

$$100 : \underline{20 :: 60}$$
$$12|00;\ \text{then, } 60 + 12 = 72 \text{ cents.}$$

Here, \$100 (regarded as first cost): \$20 (the gain on \$100) :: 60 cents (the first cost) : 12 cents, the gain on 60 cents; then 60 plus 12 = 72 cents, the required selling price. (By cancellation, we have 60 ÷ 5 = 12.) Or, say: *As \$100 is to \$100 plus the gain, or minus the loss per cent., so is the cost to the selling price.* Thus, taking the same example, we have:

$$100 : \underline{120 :: 60}$$
$$72|00$$

Here, what cost \$100 is sold for \$120; and what cost 60 cents will be sold, in proportion, for 72 cents.

RULE III. *To find the first cost from the gain per cent., and the selling price:* As \$100 plus the gain, or minus the loss per cent. is to \$100, so is the selling price to the first cost.

EXAM. 3. Sold 12 musical instruments for \$1500, and gained 25%, what was the first cost of each?

$$125 : \underline{100 :: 1500}$$
$$\underline{8} \quad \underline{150000}$$
$$1000 \quad 1200|000 \div 12 = \$100$$

Here, the analogy is simply this: As the selling price, $125, is to $100 (considered cost), so is $1500, the selling price, to the cost price ($1200).

To divide by 125, the dividend and divisor are multiplied by 8, and the new divisor is 1000, by which we divide, getting $1200, the cost price of 12 instruments. Dividing this by 12 gives $100, the cost price of each.

QUESTIONS WITH THEIR SOLUTIONS.

1. In closing the Ledger at the end of a year, the Dr. side of the Mdse. account is $38750, and the Cr. side $46500; what is the gain per cent.?

The Dr. side of Mdse. represents the purchases, or cost price, and the Cr. side, the sales, or selling price; the difference of the two will be the gain or loss on the account.

$$\overset{\text{Sales.}}{46500} - \overset{\text{Purchase.}}{38750} = 7750 \text{ gain.}$$

$$\begin{array}{l} 38750 : \underline{7750 :: 100} \\ \,)\ 775000\ (20\% \\ \underline{775000} \end{array}$$

Here, we find the gain on the account to be $7750; then as the cost price is to the gain on that price, so is $100 to the gain per cent. (20%).

2. A merchant sold 24 musical instruments for $125 each; he gained 25% on half, and lost 25% on the remainder; did he gain or lose on the transaction, and how much?

ANALYSIS: 12 at \$125 each = \$1500; then \$100 plus the gain, \$25, or \$125, the selling price, is to \$100 (considered cost), as \$1500, the selling price, to its corresponding cost; thus:

$$\begin{array}{rl} 125 : & \underline{100 :: 1500} \\ \underline{8} & \underline{150000} \\ 1000) & 1200|000 \end{array}$$

Then, \$1500, the selling price, minus \$1200 cost = \$300 gain on the first half.

$$\begin{array}{rl} \text{Now, } 100 - 25\text{, or, } 75 : & \underline{100 :: 1500} \\ \underline{4} & \underline{150000} \\ 3|00) & \underline{6000|00} \\ & \$2000 \text{ cost price.} \end{array}$$

Then, \$2000 cost minus \$1500 selling price = \$500 loss on the remaining half.

The loss on the transaction is		\$500
The gain " "		300
	Therefore he lost	\$200

3. A music dealer paid \$1500 for 12 musical instru ments which he wished to sell at a profit of 20%; what must he charge for each instrument?

$$\begin{array}{rl} 100 : & 120 :: 1500 \\ 12 : & 1 \end{array}$$

This is a simple question in Compound Proportion, in which \$100, considered as cost, is to \$120, the selling price, as \$1500

cost is to its corresponding selling price; and next, as 12 articles is to 1 so is the price of 12 to the price of 1; the said proportion, by cancellation, becoming:

$$10 : 1 :: 1500$$

and the fourth proportional \$150, or the price which must be charged for each instrument.

NOTE. — From this we see that when the number of articles is 12, and the gain per cent. desired to be made is 20, we get the price of a single article, *including* 20 *per cent.*, by simply dividing the price of 12 by 10. And to divide a number by 10 we simply cut off one figure from the right of the dividend; in other words, move the decimal point one place to the left. Hence,

When goods are bought or sold by the dozen we can readily tell what each article must be sold for so as to make 20 per cent. profit on the sale; and from this, by the method of aliquot parts (see page 109), we can get the price at any rate per cent., as illustrated in the following:

4. If a dozen silk hats be bought for \$54, what must each be sold for to make 40% profit?

$$\begin{array}{r} 100 : 140 :: 54 \\ 75|60 \div 12 \\ \$6|30 \end{array}$$

The process, by the regular method, would be: As \$100, considered cost, is to \$140, the selling price, so is \$54 cost, to its corresponding selling price, \$75.60. Dividing this by 12, gives \$6.30, the price at which each hat must be sold to make 40%. Or thus:

Dividing \$54 by 10, gives \$5.40, the price of one hat, including 20% profit, or the same as if the selling price were \$120 (\$100 being considered cost). This lacks 20% of the desired profit.

20% is $\frac{1}{6}$ of 120; now, by simply adding $\frac{1}{6}$ of $5.40 to itself, we get the price at 40%, thus:

$$\begin{array}{r@{\;=\;}r} \$5.40 & 120 \\ 90 & 20 \\ \hline \$6.30 & 140 \end{array}$$

NOTE.— A little practice will enable a person to solve such questions mentally by this process.

5. Suppose every thing as in the last question, only 10% was made by the sale, instead of 40; what was the selling price of each hat?

$$\begin{array}{r@{\;=\;}r} \$5.40 & 120 \\ 45 & 10 \\ \hline \$4.95 & 110 \end{array}$$

10% is $\frac{1}{12}$ of 120; subtract, the selling price is $4.95.

Questions of the following nature will be found useful, and, from the examples and illustrations already given, will be readily understood:

6. The population of Albany was 69422 in the year 1870, and 90905 in 1880; what was the rate per cent. of the increase during the interval?

ANALYSIS: By taking the difference we find 21483, the increase of population. Then,

As 69422 : 21483 :: 100 : 30.94+, required rate.

7. Between 1850 and 1870 the population of Albany increased by 36.76 per cent., and in the latter year it was 69422; what was it in 1850?

136.76 : 100 :: 69422 : 50762, nearly, the population in 1850.

DIVISION INTO PROPORTIONAL PARTS.

Rule. *To divide a given quantity into parts which shall be proportional to given numbers:* As the sum of the given numbers is to any one of them, so is the entire quantity to be divided to the part corresponding to the number used as the second term of the proportion.

Exam. 1. Proof spirits are composed of 48 parts of alcohol, or pure spirit, and 52 of water; how much of each is contained in 40 gallons of proof spirits?

$$(48 + 52 = 100);\ \text{then, } 100 : 48 :: 40 : 19.20 \text{ alcohol.}$$
$$100 : 52 :: 40 : 20.80 \text{ water.}$$

Or, having found 19.20 alcohol, deduct it from 40; the difference is 20.80 water, as found by the analogy.

Exam. 2. Suppose a train to start from Albany to New York, going at the rate of 20 miles an hour, and another at the same time from New York to Albany, going 30 miles an hour; where will they meet, the distance between the two places being 145 miles?

$(20 + 30 = 50)$; then $50 : 20 :: 145 : 58$ miles from Albany. Or, by taking the train from New York: $50 : 30 :: 145 : 87$ miles from New York.

The operation is proved to be correct by adding the results together.

EXAM. 3. Divide \$9500 among father, mother and son in such a manner that the father's share may be one-half greater than the mother's, and the mother's one-half greater than the son's.

ANALYSIS: Here the parts are evidently 1, $1\frac{1}{2}$ and $2\frac{1}{4}$, or 4, 6 and 9. Then

$(4 + 6 + 9 = 19)$ $19 : 4 :: 9500 : 2000$, son's share.
$19 : 6 :: 9500 : 3000$, mother's share.
$19 : 9 :: 9500 : 4500$, father's share.
\$9500

EXAM. 4. A quantity of flax seed being converted into oil, the result was found to be 329 pounds of oil and 640 pounds of cake; how much oil is that to the bushel, and what per cent.; a bushel of seed being 56 pounds, and a gallon of oil $7\frac{1}{2}$ pounds?

$(329 + 640 = 969)$; then $969 : 329 :: 56 : 19.013$, the number of pounds of oil to 56 pounds, or, to the bushel.

Now, $19.013 \div 7\frac{1}{2} = 2.535$, or $2\frac{1}{2}$ gallons nearly.

Next, $969 : 329 :: 100 : 33.952$, or 34% nearly.

EXAM. 5. Pure water is composed of oxygen and

hydrogen, in such proportions that the weight of the former is to that of the latter as 15 to 2. Required the weight of each contained in a cubic foot, or 1000 ounces, avoirdupois weight of water.

$$(15 + 2 = 17);\ \text{then } 17 : 15 :: 1000 : 882\tfrac{6}{17} \text{ oz. oxygen.}$$
$$17 : 2 :: 1000 : \underline{117\tfrac{11}{17}} \text{ oz. hydrogen.}$$
$$1000 \text{ ounces.}$$

$$\begin{array}{r|l}
\multicolumn{2}{l}{15000 \div 17} \\
900 & 00 \div 102 \\
18 & 00 \\
\hline
882 & \frac{36}{102} = \frac{6}{17}
\end{array}$$

To divide the product of the second and third terms of the first proportion, or 15000, by 17, we multiply both by 6 (seeing that 17 is nearly *one-sixth* of 100). Then dividing 90000 by 102 (Rule II, Division), we get $882\frac{36}{102}$ ounces oxygen, the fraction $\frac{36}{102}$ being equal to $\frac{6}{17}$. Now, $1000 - 882\frac{6}{17} = 117\frac{11}{17}$ ounces hydrogen. Or thus:

$$\begin{array}{r|l}
120 & 00 \div 102 \\
2 & 40 \\
\hline
117 & 60 \\
 & 6 \\
\cline{2-2}
 & 66 \div 6 = 11, \text{ that is, } \frac{11}{17}.
\end{array}$$

Multiplying 17 and 2000 (2×1000), each by 6, we have 12000 to be divided by 102. The quotient is 117, and the remainder 66, which is divided by 6 (by which we multiplied), and the *true* remainder is 11, that is, when fully expressed, $\frac{11}{17}$; or, $\frac{66}{102} = \frac{11}{17}$.

PARTNERSHIP.

Case I.

The gains and losses of partners in business may be ascertained as in the last rule by the following proportion: *As the whole stock is to the whole gain or loss, so is the stock of any partner to his gain or loss.*

Exam. 1. A. and B. form a copartnership; A. furnishes \$5000 and B. \$7000 as capital; they gain \$960; what is each man's share of the gain?

$$(5000 + 7000 = 12000);\ \text{then,}$$

$$12|000 : \underline{5000 :: 960}$$

$$\underline{4800|000}$$

$$\$400,\ \text{A.'s gain.}$$

B.'s gain is found by the same analogy, using 7000 for the second term.

Exam. 2. Two brothers, John and James, purchase a house jointly for \$25000; John contributed \$10000 and James \$15000 of the purchase-money. They let the house for the yearly rent of \$2000; what share of the rent is each to receive?

25000 : 2000 :: 10000 : 800, John's share.
25000 : 2000 :: 15000 : 1200, James' share.
$2000

CASE II.

To find each partner's share of the gain or loss when their capital is employed for *unequal* periods of time.

RULE. *Multiply each stock by the time of its continuance in trade; then, using the products as stocks, proceed according to Case I.*

EXAM. 1. A. and B. form a partnership. A. contributes $3500 for 12 months, and B. $4500 for 9 months. They gain $1600; what is the share of each?

$$\begin{array}{r} 3500 \times 12 = 42000 \\ 4500 \times \ \ 9 = 40500 \\ \hline 82500 \end{array}$$

Then, as $82500 : $1600 :: $42000 : $814.54, A.'s gain.
82500 : 1600 :: 40500 : 785.46, B.'s gain.

The reason of the process will be evident from the consideration, that $3500 for 12 months is equivalent to 12 times that for 1 month, that is, to $42000; and $4500 for 9 months is equivalent to 9 times that for 1 month, that is, to $40500. Hence, if these increased stocks be employed, it is evident that, since the times are then to be regarded as equal, the process will be the same as in Case I.

NOTE.—It need scarcely be remarked that the times, in all such operations, must be of the same denomination. If, for instance, one was 12 weeks, and the other 9 months in the foregoing example, the 9 months should be reduced to weeks, or the 12 weeks to months.

BANKRUPTCY.

The estate of a bankrupt may be divided among his creditors by the following analogy: *As the sum of all the claims on the estate is to the value of the whole estate, so is the claim of any creditor to his dividend or share.*

EXAM. 1. A bankrupt owes A. \$350, B. \$650 and C. \$1500. His whole estate is worth only \$1500; what is the share of each creditor?

$$(\$350 + \$650 + \$1500 = \$2500)$$

Then, as \$2500 : 1500 :: 350 : 210, A.'s share.
2500 : 1500 :: 650 : 390, B.'s share.
2500 : 1500 :: 1500 : 900, C.'s share.
\$1500 = the whole estate.

NOTE.—In the division of a bankrupt's estate, it is usual first to find *how much on the dollar* he can pay; that is, how much the creditors will receive for each dollar of their respective claims. Thus, resuming the same example, we have this analogy or proportion:

$$\frac{25|00}{1|00} : \frac{1500}{1500|00} :: \$1, \text{ or 100 cents.}$$
$$60|00$$

The sum of all the claims is to the whole estate as \$1, or 100 cents, to the proportional part, corresponding to a dollar, which is found to be 60 cents, or 60 per cent. Then 60% of \$350 = \$210, A.'s share, as before. And B.'s and C.'s can be found in like manner.

USEFUL RULES.

RULE I. *To find the rate at which a given principal will gain a certain interest in a stated time:*

$$\left.\begin{array}{lcl}\text{As the given principal} & : & \$100 \\ \text{the stated time} & : & 1\text{ year}\end{array}\right\} :: \text{the interest} : \text{the rate.}$$

EXAM. If $5000, invested for 1 year and 6 months, gain $525; what is the rate per cent.?

The problem fully expressed is this: If $5000, in 18 mo. gain $525; what will $100 gain in 12 mo.; and for this we have the following proportion:

$$\left.\begin{array}{lcl}\text{As } \$5000 & : & \$100 \\ 18\text{ mo.} & : & 12\text{ mo.}\end{array}\right\} :: \$525 : \text{the rate.}$$

Reducing this to a simple proportion, we have the following: 90000 : 1200 : : $525 : x, or the rate. Multiplying the second and third terms, now, and dividing by the first, we get the rate, thus:

$$\frac{1200 \times 525}{90000} = 630000 \div 90000 = 7\%, \text{ the rate.}$$

Hence, if we draw a vertical line and set the given principal and the stated time on the left; and on the right, set the given interest, $100 and 1 year (or 12 mo. as the case may be); then divide the product of the numbers on the right by the product of those on the left; the quotient is the rate.

$$\begin{array}{r|l} 5000 & 525 \\ 18 & 100 \\ & 12 \\ \hline 90000 & 630000(7 \end{array}$$

NOTE.—It may be well to remark that, in all cases where the terms will admit, the work can be cut down by cancellation.

RULE II. *To find what principal, in a stated time, will gain a certain interest, at a given rate per cent.:*

$$\left.\begin{array}{lcl}\text{As the given rate} & : & \$100 \\ \text{the stated time} & : & 1 \text{ year}\end{array}\right\} : : \text{the interest} : \text{the principal.}$$

EXAM. What sum of money must be invested to gain $525 in 1 year and 6 months, at 7 per cent.?

The problem fully expressed is this: If $100 gain $7 (7%) in 12 months; what sum will gain $525 in 18 months; and for this we have the following proportion:

$$\left.\begin{array}{lcl}\text{As } \$7 & : & \$100 \\ 18 \text{ mo.} & : & 12 \text{ mo.}\end{array}\right\} : : \$525 : \text{the principal.}$$

And by reduction to a simple proportion, we have the following:

As 126 : 1200 : : 525 : x, or the principal.

Multiplying the second and third terms, and dividing by the first, we have: $\frac{1200 \times 525}{126} = 630000 \div 126 = \$5000$, the principal.

Or, drawing a vertical line, setting the given rate and the stated time on the left; and the given interest, $100 and 12 mo. on the right, and dividing the product of these on the right by the product of those on the left, we get the principal.

$$\begin{array}{r|r} 7 & 525 \\ 18 & 100 \\ & 12 \\ \hline 126 & 630000 \\ \hline & \$5000 \end{array}$$

RULE III. *To find the time in which, at a given rate per cent. per annum, a given principal will produce a certain interest:*

$$\left.\begin{array}{lcl}\text{As the principal} & : & \$100 \\ \text{the rate} & : & \text{the interest}\end{array}\right\} : : 1 \text{ year} : \text{to the time.}$$

EXAM. How long will it take to have $5000 gain $525, at 7 per cent. per annum, simple interest?

The problem fully expressed is this: If $100, in 1 year, gain $7 (7%) how long will it take $5000 to gain $525; and for this we have the following proportion:

$$\text{As } \left.\begin{array}{r}\$5000 : \$100 \\ \$7 : \$525\end{array}\right\} :: 1 \text{ year} : \text{the time required.}$$

Reducing this to a simple proportion, we have the following:

As 35000 : 52500 : : 1 : x, or the time.

And we have, now, simply, 52500 to be divided by 35000. Cutting off the three ciphers in 35000 and three places from 52500 (this divides each by 1000, and does not alter the proportion) we have 52.5 to be divided by 35, as shown in the margin.

To divide by 35, we use the component factors, 5 and 7 ($5 \times 7 = 35$) the quotient is 1.5 years, or 1 yr., 6 mo.

5	52.5
7	10.5
	1.5 years

Now, the interest on $5000 for 1 year @ 7%, is $350; and the given interest is $525, and if 525 be divided by 350 the quotient is the time; hence, the rule may be stated as follows:

Divide the given interest by the interest on the principal for 1 *year, at the given rate, and the quotient is the time in years and decimals.*

RULE IV. *To find what principal, in a stated time, will increase to a given amount, at a given rate per cent. per annum:*

This is the same as *finding the true present worth of a debt*, and the rule given on page 133, will answer here, also, viz.:

As $100 plus its interest for the given time, and at the given rate, is to $100, so is the given amount to the principal sought; illustrated in the following:

EXAM. What principal, in 1 year and 6 months, at 7 per cent., simple interest, will amount to $5525? Or, in other words, what is the true present worth of a debt of $5525, due in 1 year and 6 months, at 7 per cent.?

Here, the interest of $100 for 1 year and 6 months, at 7%, is $10.50. Then, $100 + $10.50 = $110.50, the *amount* of $100 for the given time and at the given rate; and applying the foregoing rule, we have the following proportion:

As \$110.50 : \$100 :: \$5525 : x, or the required principal. Throwing off the decimal, now, in the first term, we have the following: $11050 : 10000 :: 5525 = \frac{10000 \times 5525}{11050} = 55250000 \div 11050 = \$5000$, the required principal, or, the true present worth of the debt.

Questions like the following, and which are of a useful kind, are of the *same nature* as those regarding the interest of money:

EXAM. If the population of a city was 240000 in the year 1896, and 300000, in 1906; what was the rate per cent. of increase during the interval?

By taking the difference of these we find 60000, the increase of population. The question now is: If 240000 gain 60000; what will 100 gain; and for this we have the following proportion:

As 240000 : 60000 :: 100 : x, or the rate; that is: $\frac{60000 \times 100}{240000} = 6000000 \div 240000 = 25$ per cent., the rate.

EXAM. Between 1896 and 1906, the population increased 25 per cent., and in the latter year it was 300000; what was it in 1896?

In this, $100 + 25 = 125$: Then, we have 125 : 100 :: 300000 : x; and this gives us: $\frac{100 \times 300000}{125} = 30000000 \div 125 = 240000$, the population in 1896. (*See examples, page* 148.)

EXAM. If \$25000 be invested in property which rents for \$3500 a year, and on which \$500 are paid in taxes; what rate of interest does the investment pay?

Here, $\$3500 - \$500 = \$3000$, the income from \$25000. Now, \$25000 : \$3000 :: \$100 : rate; and we have $\frac{3000 \times 100}{25000} = 300000 \div 25000$, or, cutting off three ciphers from each, we have $300 \div 25 = 12\%$, the rate.

PRACTICAL HINTS FOR BUILDERS.

Lumber and *sawed timber*, as plank, scantling, etc., are usually estimated in *Board Measure*, *hewn* and *round timber* in *cubic* measure.

A board foot is 12 inches long, 12 inches wide and 1 inch thick; in other words, it is *a square foot* 1 *inch thick*.

In *board measure* all boards are assumed to be 1 inch thick.

NOTE. — Lumber 1 inch thick or less is sold by surface measure, and in the trade is denominated *boards*. If more than 1 inch thick it is called *plank*, and is computed at 1 inch thickness, or standard thickness; that is, the product of the surface measure in square feet multiplied by the thickness in inches is the number of feet of lumber at board measure.

RULE. *To find the number of feet, board measure, in a board or plank:* Multiply the length in feet by the width in inches and divide the product by 12; the result is the number of feet at 1 inch, or the standard thickness. Next, multiply the result thus found by the thickness of the plank in inches; the product is the number of feet of standard thickness, or board measure.

EXAM. 1. How many feet board measure in a piece of pine lumber 16 feet long, 9 inches wide and $1\frac{1}{4}$ inches thick?

ANALYSIS: $16 \times 9 = 144$; then, $144 \div 12 =$ 12 feet at 1 inch thick.

Then, $12 \times 1\frac{1}{4} = 15$ ft.; or simply add $\frac{1}{4}$ of 12, or 3
and we have the number of board feet = 15 feet.

If the plank were $1\frac{1}{2}$ inches thick we would add the $\frac{1}{2}$ of 12, or 6, making 18 feet, board measure; if $2\frac{1}{4}$ inches thick, it would be $2\frac{1}{4}$ times 12, or 27 feet, etc.

NOTE.—When the piece is 12 feet long and 1 inch thick, or less, the surface feet will be the same as the width in inches; thus, a board 12 feet long, 7 inches wide and $\frac{1}{2}$ inch thick is 7 feet, surface measure.

In the trade it is customary to have $1\frac{1}{4}$ inch lumber resawed into boards about $\frac{1}{2}$ inch thick, commonly called panel-stuff, which is bought and sold by surface measure as if it were inch, or standard thickness; but the price is reduced accordingly. For instance, $1\frac{1}{4}$ inch pine lumber, worth \$50 per thousand feet, would, when resawed, be sold for about \$32 to \$35 per thousand feet.

Hence, questions of the following nature are frequently asked:

EXAM. 2. Whether is it more advantageous to buy 1000 feet of $1\frac{1}{4}$ inch pine at \$50, and have it resawed into panel-stuff, paying \$2 for the sawing, or to buy the same quantity already resawed, at \$35 per thousand?

1000 feet of $1\frac{1}{4}$ inch @ \$50, and \$2 for sawing = \$52.

In 1000 feet of $1\frac{1}{4}$ inch measurement there are 800 feet at 1 inch, or surface measurement, found thus:

$$\begin{array}{rl} & 1000 \text{ feet of } 1\frac{1}{4} \\ \text{less one-fifth } (\frac{1}{5}) & \underline{\ 200\ } \\ & 800 \text{ feet.} \end{array}$$

There are 5 quarters in $1\frac{1}{4}$, therefore, 1 quarter ($\frac{1}{4}$) is the fifth of 5; deducting this leaves the 1 inch, or surface. Now, since

every piece of $1\frac{1}{4}$ inch makes 2 pieces when sawed, 800 feet is doubled, giving 1600 feet of panel. Then 1600 feet at \$35 per thousand gives \$56, or \$4 more than \$52, as found above. It would be more advantageous, therefore, to buy at \$50 and pay \$2 for sawing.

EXAM. 3. A carpenter wishing to get 1000 feet of 1-inch pine boards dressed to $\frac{3}{4}$ inch, and not getting the quality suitable for his purpose in the market, concluded to take $1\frac{1}{2}$-inch lumber and have it resawed; how much of the latter did he require?

To solve problems of this nature, it must be borne in mind that the *surface* is the same regardless of the thickness.

But 1000 feet surface is 1500 feet, standard measure, at $1\frac{1}{2}$ inch thick, and since each piece which goes to make 1500 feet will make two pieces when resawed, it follows that *half* the quantity will make 1000 feet surface.

Dividing 1500 feet then by 2 gives 750 feet of $1\frac{1}{2}$-inch lumber, the required quantity.

PROOF: 750 feet $1\frac{1}{2}$-inch lumber,

less $\frac{1}{3}$ 250 (1 half inch = $\frac{1}{3}$ of 3 half inches, or $1\frac{1}{2}$ inch)

500 feet at 1 inch thick, and doubling this gives 1000 feet surface; that is, 750 feet of $1\frac{1}{2}$ inch resawed.

Or the problem might be solved thus: take half 1000 feet surface and we have 500; to this add one-half and we have 750 feet of $1\frac{1}{2}$ inch as before. Adding one-half of 500 feet to itself, we need scarcely remark, is multiplying 500 feet surface by $1\frac{1}{2}$, the thickness.

SOUTHERN PINE.

Southern pine, commonly called "Georgia pine," or "yellow pine," comes in various lengths and widths, and it not unfrequently happens that builders requiring a quantity of flooring or

ceiling will give a hurried order to the dealer for such quantity; and the dealer, in delivering the same from the mill, where it has probably just been dressed, will not spend the time to get the standard measurement, but, instead, will merely take the lengths of the pieces, or the lineal feet, converting the same at leisure into standard, or board measure. For such emergencies we give the following simple

RULE. *To find the lineal feet for any surface and to convert the same into standard or board measure:* Multiply the surface to be covered by 12, and divide the product by the width of the board or plank; the quotient will be the lineal feet. Next, reverse the process; that is, multiply the lineal feet by the width of the board or plank (in the rough, or before being dressed), and divide by 12; the result is surface feet. Then add for the extra thickness, if more than 1 inch thick, and we have the standard, or board measure.

EXAM. 1. How many lineal feet of $1\frac{1}{4}$ by 4 inches (face) of Southern pine will cover 2550 feet surface?

$$2550 \times 12 \div 4 = 7650 \text{ lineal feet.}$$

Reason of the rule: When a board or plank is 12 feet long it will cover as many feet surface as there are inches in the width of the face of such board or plank. Now, if a piece 4 inches on the face and 12 feet long cover 4 feet surface, how many feet in length will cover 2550 feet surface? And for this we have the following proportion, or analogy: 4 : 2550 :: 12; that is, 4 feet surface is to any given surface (2550 in this case) as 12 feet (the length corresponding to 4 feet surface) is to the length, or lineal feet, corresponding to 2550 feet surface. Multiplying the second and third terms and dividing by the first, we get 7650 feet in length, or lineal feet.

Next, how many feet, standard, or board measure, in 7650 lineal feet of $1\frac{1}{4}$-inch flooring, 4 inches on the face?

A piece which gives 4 inches (face) when dressed, is usually $4\frac{1}{2}$ inches in the rough; that is, before being dressed, and, as *it is customary to buy and sell at what the lumber measures in the rough*, we multiply by $4\frac{1}{2}$ instead of 4, thus:

$$\frac{7650 \times 4\frac{1}{2}}{12} = 2868 \text{ feet, the surface,}$$

then adding $\frac{1}{4}$ (the thickness over 1 inch) or $\underline{717,}$ we find the standard, or board measure to be 3585 feet.
Hence, in

MAKING ESTIMATES

for flooring, ceiling, etc., the calculations for the material should be made on the measurements in the *rough*.

EXAM. 2. How many feet of "Georgia pine" flooring, $2\frac{1}{2}$ inches on the face and $1\frac{1}{4}$ inches thick, required to cover 3 floors 40×36 feet?

$$40 \times 36 \times 3 = 4320 \text{ feet, the surface to be covered.}$$

Now, a piece of flooring $2\frac{1}{2}$ inches on the face and 12 feet long (regardless of the thickness) *will cover* $2\frac{1}{2}$ *feet surface*. But a piece of flooring $2\frac{1}{2}$ inches (face), 12 feet long, was 3 inches (face) in the rough, or 3 feet surface, and at $1\frac{1}{4}$ inches thick it was $3\frac{3}{4}$ feet, board measure.

The question now is: if $3\frac{3}{4}$ feet, board measure, cover $2\frac{1}{2}$ feet surface, how many feet, board measure, will cover 4320 feet surface? And the proportion is:

$$2\tfrac{1}{2} : 4320 :: 3\tfrac{3}{4}$$

Or, reducing the first and second terms to the same denomination, halves, and the third to quarters, or fourths,

we have $5 : 8640 :: 15$; and by cancellation
this becomes $1 : 8640 :: 3$.

Multiplying 8640 by 3, now, gives 25920 fourths, or quarter feet; which, being divided by 4, gives 6480 feet, board measure, in the rough, or the number of feet to be paid for. From this, it will be seen that, *when the material is narrow, it takes about* $1\frac{1}{3}$ *times the surface to be covered, for the required number of feet, board measure, at* $1\frac{1}{4}$ *inches thick.*

EXAM. 3. A builder requires $\frac{1}{2}$ inch pine ceiling, $2\frac{1}{2}$ inches on the face, to cover 10000 feet surface. He can buy ceiling suitable for his purpose for \$30 per thousand feet, surface measure, or, $1\frac{1}{4}$ inch by 6 inch pine, from which to make such ceiling, for \$35 per thousand feet. Which is the more profitable, the cost for making each piece of ceiling from the $1\frac{1}{4}$ inch being $2\frac{1}{2}$ cents?

ANALYSIS: A piece $2\frac{1}{2}$ inches on face, 12 feet long (regardless of thickness), will cover $2\frac{1}{2}$ feet surface. Dividing 10000 feet by $2\frac{1}{2}$, or using their doubles, $20000 \div 5$, gives 4000 pieces at $2\frac{1}{2}$ feet each. But a piece $2\frac{1}{2}$ feet surface, dressed, was 3 feet in the rough, and, therefore, 4000 pieces at 3 feet each, would make 12000 feet surface measure, or what has to be paid for. This, at \$30 per thousand, is \$360, the cost.

Next, a piece $1\frac{1}{4}$ by 6 inches and 12 feet long contains 6 feet and the quarter of 6, or $7\frac{1}{2}$ feet, standard measure. It will take 1000 pieces of $1\frac{1}{4} \times 6$ (each piece makes 4 pieces of ceiling when milled) to make 4000 pieces of ceiling. Now, 1000 pieces at $7\frac{1}{2}$ feet each makes 7500 feet, standard or board measure, and at \$35 per thousand this gives \$262.50. Adding to this the milling of 4000 pieces at $2\frac{1}{2}$ cents each, or \$100, we have \$362.50, the cost. It is more profitable to buy the ceiling already made, in this case.

NOTE. — From the three foregoing examples it will be seen that, by taking 12 *feet in length as a basis of calculation*, estimates for flooring, ceiling, etc., can be readily made.

ROOF ELEVATIONS, ETC.

By the "pitch" of the roof is meant the ratio which the height of the ridge above the level of the roof-plates bears to the span, or the distance between the *supports* or studs on which the roof rests.

The usual pitches are the *Common* or true pitch, in which the rafters are three-fourths of the width of the building; the *Gothic* pitch, in which the length of the principal rafters is equal to the width of the building; the *Pediment* pitch is when the perpendicular height is $\frac{2}{9}$ of the width. There are also the $\frac{1}{4}$ pitch, $\frac{1}{8}$ pitch, $\frac{3}{8}$ pitch, etc.

RULE. *To find the length of rafter for any particular pitch of roof:* To the square of the perpendicular height of roof add the square of half the width of the building, the square root of the sum is the length of the rafter.

NOTE.— The method of extracting the square root is given in almost any common-school arithmetic.

It need scarcely be remarked that the rafters for the Common pitch, and also for the Gothic, are obtained without the aid of this rule.

SHINGLES, LATH, ETC.

A "*shingle*" is 4 inches wide and from 16 to 18 inches long. But shingles are seldom made of a uniform width; they vary from 2 to 10 inches, more or less, and are put up in bundles, or bunches, containing 250 shingles each (not by count but on an average of 4 inches to a shingle). Hence, there are 4 bunches to 1000 shingles.

Since a shingle is reckoned at 4 inches, it is evident that the number of shingles required to cover a roof will depend on how much of the shingle is "laid to the weather." Thus, if 6 inches be laid to the weather, a shingle will cover 24 square inches (4×6); and by dividing 144 square inches (1 square foot) by 24, we find it will take 6 shingles to cover 1 square foot.

Again, if laid 5 inches to the weather, a shingle will cover 20 square inches (4×5); and dividing 144 by 20, gives $7\frac{1}{5}$ shingles to the square foot.

Hence, it will be seen that, by multiplying the number of square feet to be shingled, in the one case, by 6, and by $7\frac{1}{5}$ in the other, we get the number of shingles required at 6 inches and 5 inches, respectively, to the weather.

Now, shingles are generally laid from 5 to $5\frac{1}{2}$ inches to the weather, and for *practical purposes* the following simple rule will be found sufficiently accurate: *Multiply the number of square feet to be shingled by* 7; *the product is the number of shingles required, nearly.*

EXAM. How many sawed pine shingles required to cover a building 50 feet in length and 36 feet in width, the roof being of the common or true pitch?

ANALYSIS: In the true pitch the rafter is $\frac{3}{4}$ of the width of the building; $\frac{3}{4}$ of $36 = 27$ feet, the length of the rafter. Then, 27 doubled and multiplied by 50 will give the surface, or the number of square feet to be shingled, thus:

$27 \times 2 \times 50 = 2700$ feet, the surface of roof; and $2700 \times 7 =$ 18,900 shingles, the number required.

LATH.

NOTE. — It is customary among the dealers to make use of the *singular form*, lath; as, "Have you got any lath?" "How many lath will cover 1000 feet?" etc.

A *lath* is 4 feet long, $1\frac{1}{4}$ to $1\frac{1}{2}$ inches wide and about $\frac{3}{8}$ inch thick, usually made from pine, spruce or hemlock.

Lath are seldom, if ever, counted in bunching; they are generally put into a gauge, or measure, which contains about 100 pieces, more or less, and tied in a bunch; hence, 10 bunches make 1000 lath.

The surface of a lath 4 feet, or 48 inches long, and $1\frac{1}{2}$ inches wide, is $48 \times 1\frac{1}{2}$, or 72 square inches, and of 2 lath, 72×2, or 144 square inches: therefore, 2 lath, $1\frac{1}{2}$ inches, set edge to edge, will cover a square foot. Hence the following simple

RULE. *To find the number of lath required to cover any surface:* Multiply the number of square feet to be lathed by 2; the product is the number required.

EXAM. How many lath will be required for a room 24 feet long, 20 feet wide, and 9 feet 6 inches high?

ANALYSIS: The length of the four walls is $(24 + 24 + 20 + 20)$ 88 feet; then $88 \times 9\frac{1}{2}$ (the height 9 feet 6 inches) $=$ 836 square feet, or surface of walls. Next, 24×20 (the ceiling) $=$ 480 square feet or surface of ceiling. Putting the two surfaces together we get .... 1316, the number of square feet to be lathed. Doubling this, we have 2632, the number of lath required.

NOTES.—1. Lath are usually set about $\frac{1}{4}$ inch apart, to allow for the "clinch"; hence, when $1\frac{1}{2}$ inch lath are used, a deduction of about one-tenth, or 1 bunch in every 10, may be made. Thus, in the foregoing example, 2632 less 263 ($\frac{1}{10}$ of 2632) $=$ 2369, would be nearest the true result. But for $1\frac{1}{4}$ inch lath no deduction is necessary.

2. Allowance must, of course, be made for doors, windows, etc.

CONSTRUCTION AND CAPACITY OF BINS, ETC.

The *Standard Bushel* of the United States is a cylindrical measure $18\frac{1}{2}$ inches in diameter and 8 inches deep, and contains 2150.42 cubic inches. (The capacity, 2150.42, is found by multiplying the square of the diameter, $18\frac{1}{2}$, or 18.5, by .7854, and the product by the depth, 8 inches.)

Since a cubic foot contains 1728 cubic inches, and a standard bushel contains 2150.42 cubic inches, a bushel is equal to $1\frac{1}{4}$ cubic feet, nearly ($2150 \div 1728 = 1\frac{1}{4}$, nearly), the proportion being 1 to $1\frac{1}{4}$, or 4 to 5, nearly. Hence,

To find the capacity of a bin in bushels: Add $\frac{1}{4}$ of the quantity in bushels to itself; the sum will represent the capacity of the bin.

Thus, what must be the capacity of a bin to contain 160 bushels of wheat? $160 + 40$ ($\frac{1}{4}$ of 160) $= 200$, the number of cubic feet, or capacity of bin. And

To find the number of bushels contained in a bin: Deduct $\frac{1}{5}$ of the capacity of the bin from itself; the remainder will represent the number of bushels.

Thus, how many bushels of wheat in a bin of 200 cubic feet capacity?

$200 - 40$ ($\frac{1}{5}$ of 200) $= 160$ bushels of wheat in a bin of 200 cubic feet. Hence,

Any two dimensions of a bin being given, the third dimension can be found, thus:

(1) Increase the number of bushels by $\frac{1}{4}$ of itself; the result will represent the number of cubic feet contained in the bin. (2) Divide the contents in cubic feet by the product of the two dimensions, and the quotient will be the other dimension.

EXAM. 1. What must be the depth of a bin to contain 280 bushels, its length being 10 feet and its width 5 feet?

ANALYSIS: $280 + 70 = 350$; then $350 \div 50$ $(10 \times 5) = 7$ feet, the depth.

EXAM. 2. What is the value of a bin of wheat 20 feet long, 12 feet wide, and 5 feet deep, at $2 a bushel?

ANALYSIS: $20 \times 12 \times 5 = 1200$; then, $1200 - 240$ ($\frac{1}{5}$ of 1200) $= 960$, the number of bushels in bin. 960 at \$2 = \$1920, the value.

EXAM. 3. A coal bin is 12 feet long and 6 feet wide; how deep must it be to contain 12 tons of chestnut coal, the contents of a ton of chestnut coal being 38 cubic feet?

ANALYSIS: $38 \times 12 = 456$, the contents of 12 tons; then $456 \div 72$ $(12 \times 6) = 6\frac{1}{3}$ feet, or 6 feet 4 inches, the depth.

RULE. *To find the capacity of a vessel or space in gallons:* Divide the contents in cubic inches by 231 for liquid gallons, or by 268.8 for dry gallons.

EXAM. 1. How many gallons of water will a cistern hold that is 4 feet by 5 feet, and 6 feet deep?

ANALYSIS: $(4 \times 5 \times 6 \times 1728) \div 231 = 897\frac{51}{77}$ gallons *capacity.*

EXAM. 2. What must be the depth of a cistern that is 6 feet long and $5\frac{1}{2}$ feet wide to hold 462 gallons of water?

ANALYSIS: $\frac{462 \times 231}{1728 \times 6 \times 5\frac{1}{2}} = 1.87$ feet, the depth.

EXAM. 3. A cellar 40 feet long, 20 feet wide and 8 feet deep is half full of water. What is the cost of pumping it out at 6 cents a hogshead?

ANALYSIS: $(40 \times 20 \times 8 \times 1728) \div 231 = 47875.32$ gallons; then $47875.32 \div 63$ (gallons in a hogshead) $= 759.92$ hogsheads; and 6 times $759.92 = 4559.52$ cents. Dividing this by 2 (half the cellar) gives \$22.80, the cost.

MASONRY.

Masonry is estimated by the *cubic foot* and by the *perch*, also by the *square foot* and the *square yard*.

A perch of masonry is $16\frac{1}{2}$ feet long, $1\frac{1}{2}$ feet wide and 1 foot high. Multiplying these three dimensions together, we find there are $24\frac{3}{4}$, or 24.75, cubic feet in a perch of masonry ($16\frac{1}{2} \times 1\frac{1}{2} \times 1 = 24\frac{3}{4}$), or ($16.5 \times 1.5 \times 1 = 24.75$).

NOTE. — When stone is built into a wall without mortar or filling an allowance of $2\frac{3}{4}$ feet is made, and 22 cubic feet make a perch.

RULE. *To find the number of perches of masonry in a wall:* Divide the number of cubic feet in the work by $24\frac{3}{4}$, or 24.75; the quotient will be the number of perches.

NOTE.— Brick-layers and masons, in estimating their work by cubic measure, make no allowance for the corners of the walls of houses, cellars, etc., but estimate their work by the *girt*, that is, the entire length of the wall on the outside.

Joiners, brick-layers, and masons, make an allowance of one-half the openings or vacant spaces for doors, windows, etc.

EXAM. 1. At $4 a perch what will be the cost of building the walls of a cellar $37\frac{1}{2}$ feet long, 26 feet wide, 9 feet deep and 2 feet thick?

ANALYSIS: $75 + 52 = 127$ feet, the girt, or outside measure; then, $127 \times 9 \times 2 = 2286$ cubic feet, the solid content of walls. Next, $2286 \div 24.75 = 92.36$ perches. Multiplying this, now, by 4, gives $369.44, the cost.

NOTE.— To divide by 24.75, or $24\frac{3}{4}$, we make use of the simplified method, as pointed out at page 60, example 4, to which the reader is referred.

EXCAVATING

or digging is measured and paid for by the *cubic yard*, and *a cubic yard of earth is called a load.*

NOTE. — In a lineal yard there are 3 feet. Cubing 3 ($3 \times 3 \times 3 = 27$), we get 27 cubic feet for a cubic yard. Hence, a box 9 feet long, 3 feet wide

and 1 foot deep will contain a load of earth ($9 \times 3 \times 1 = 27$). And any two dimensions of a box to contain a load being given, the other dimension can be found, thus; Divide 27 by the product of the two given dimensions; the quotient is the other. (See example 1, page 168.)

EXAM. 2. What is the cost of digging the cellar in the last example at 50 cents a load?

ANALYSIS: $37\frac{1}{2} \times 26 \times 9 = 8775$ cubic feet of earth in cellar. Then, $8775 \div 27 = 325$ loads; and at 50 cents a load it is $162.50, the cost.

BRICK-WORK.

RULE. *To find the number of bricks in a wall:* Multiply the number of cubic feet in the work by the number of bricks in a cubic foot; the product is the number of bricks required.

NOTE.—About 22 common bricks make a cubic foot when laid.

EXAM. 1. How many common bricks in a wall 70 feet long, 20 feet high, and 12 inches thick?

ANALYSIS: $70 \times 20 \times 1 = 1400$ cubic feet in wall; then $1400 \times 22 = 30800$ bricks.

In estimating brick-laying by the square yard, the rod, or by the square of 100 feet, the work is understood to be 12 inches, or $1\frac{1}{2}$ bricks thick, which is called *standard* thickness.

RULE. *To reduce brick-laying to standard thickness:* Multiply the superficial content of the work by the number of half bricks in thickness, and divide by 3.

NOTE.—The superficial content is found by multiplying the length by the height. A rod is $16\frac{1}{2}$ feet long, consequently a square rod is 272.25 square feet ($16.5 \times 16.5 = 272.25$).

EXAM. 2. How many squares of brick-work in a wall 130 feet long, 12 feet high; the wall being $2\frac{1}{2}$ bricks thick?

ANALYSIS: $130 \times 12 = 1560$ feet, superficial content, and $1560 \times 5 = 7800$; then, $7800 \div 3 = 2600$ feet of standard work, or 26 squares ($2600 \div 100 = 26$).

LUMBER CALCULATIONS

In computing the cost of lumber, or other merchandise, at so much per thousand, the desired results are, in most cases, obtained more easily by *the method of aliquot parts,* than by the usual methods of multiplication; thus:

Take, say, 15836 feet of lumber.

At $1000 per thousand feet, the cost is at sight, or as many dollars as there are feet; and at $100, the cost is a tenth of that at $1000. At $10, the cost is a tenth of $100; at $1, a tenth of $10; at 10 cts., a tenth of $1 and at 1ct. it is a tenth of 10 cts. Hence,

$15836	=	$1000
$1583.6	=	$100
$158.36	=	$10
$15.836	=	$1
$1.5836	=	10 cts.
.15836	=	.1

In computing the cost of lumber, if we assume $100 *per thousand, as a standard price, calculations can be simplified; thus:*

EXAM. What is the cost of 15836 feet of pine lumber @ $27.75 per thousand feet?

Cutting off one figure gives the cost at $100. Then, the cost at $25 is a fourth of that, or $395.90; now, at $2.50, the cost is a tenth of that at $25, or $39.59 and at 25 cts., a tenth of that at $2.50, or $3.959. Adding the several results gives the cost at $27.75.

$1583	6	=	$100
395	9	=	$25
39	59	=	2.50
3	959	=	.25
$439	449	=	$27.75

EXAM. What is the cost of 75680 feet of black walnut at $75.75 per thousand?

In this, we have at sight the cost at $100; then, at $25, it is a fourth, or $1892, which is deducted, leaving $5676, the cost at $75. Now, 75 cts. is a hundredth part of $75, or $56.76, which is added; this gives $5732.76, the cost at $75.75.

$7568	0	=	$100
1892	0	=	$25
$5676	0	=	$75
56	76	=	.75
$5732	76	=	$75.75

EXAM. What is the cost of 75684 feet of lumber at $32.58 per thousand feet?

Here, we cut off one figure to get the cost at $100; then, one-fourth of this, or $1892.10, is the cost at $25; now, $5 is a fifth of this, or one-half of the cost at $10, got from the top line; next, $2.50 is half of $5; and for 8 cts. we take 8 times the cost at 1¢, that is, .75684, got from the top number: say 8 times .757, making use of three figures only, and allowing for those rejected; the result is $6.056.

$7568	4	=	$100
1892	1	=	25
378	42	=	5
189	21	=	2.50
6	056	=	.08
$2465	786	=	$32.58

Or thus:

Cutting off two figures gives the cost at $10, always; then, 3 times 10 are $30; now, $2.50 is a fourth of $10; and finally, 8 times .757 gives $6.056; the sum of the several results gives $2465.786, the cost at $32.58, the same as before.

$756	84	=	$10
2270	52	=	30
189	21	=	2 50
6	056	=	.08
$2465	786	=	$32.58

EXAM. What is the cost of 75684 feet, at $37.75 per thousand?

In this, the cost at $25 is one-fourth of that at $100, or $1892.10; then, $12.50 is half of $25; and 25cts. is the hundredth part of $25; the hundredth part of $1892 1 is $18.92; the sum of the several results is $2857.07, the required cost.

$7568	4	=	$100
1892	1	=	25
946	05	=	12.50
18	92	=	.25
$2857	07	=	$37.75

Or thus:

Here, we have the cost at $10, at sight, and 3 times this is the cost at $30; then, $5 is half of $10; now, $2.50 is half of $5, and 25cts. is a tenth of $2.50; the sum of the several results is the cost at $37.75, as before.

$756	84	=	$10
2270	52	=	30
378	42	=	5
189	21	=	2.50
18	92	=	.25
$2857	07	=	$37.75

CHRONOLOGICAL CALCULATIONS.

To find the weekly day for any given date from the year 1600, New Style (N. S.) for any year or century thereafter, we give the following simple

RULE I. *Subtract the centuries from the given year.*

II. *To the remainder add one-fourth of the given year; also the number of days from January* 1, *up to and including the given date, and* 1 *for every fourth century.*

III. *Divide the sum by* 7; *the remainder, counting Sunday* 1, *will be the weekly day.*

NOTES. — 1. For leap years add 1 to the centuries before subtracting always, counting 29 days in February; then proceed according to the rule.

2. When 7 is contained in the sum without remainder, Saturday is the weekly day.

3. In taking the fourth part of the given year, the remainder, if any, may be rejected; also, in taking one-fourth of the centuries.

EXAM. 1. On what day of the week did the 4th of July, 1775, happen?

SOLUTION.— Subtracting 17 centuries from the given year, we get 1758; to this we add one-fourth of 1775, rejecting the remainder, next 185, the number of days from January 1, up to and including July 4th, and 4, 1 for every fourth century in 17 (17 ÷ 4) rejecting the remainder.

1775	Jan.	31	da.
17	Feb.	28	"
1758	Mch.	31	"
443	April	30	"
185	May	31	"
4	June	30	"
7)2390	July	4	"
341—3		185	

Dividing the sum, 2390, by 7 gives a remainder of 3; then counting Sunday 1, Monday 2, Tuesday 3; we find that July 4th happened on the third day of the week, Tuesday.

EXAM. 2. On what day of the week will Washington's birthday (Feb. 22) happen in 1912?

In this, 1912 being a leap year, we subtract 20 from the given year, the remainder is 1892. Adding to this 478, the fourth part of 1912; then 53 days, the number from January 1 to February 22, and 4 (19 ÷ 4) 1 for every fourth century in 19 centuries, and dividing the sum by 7, we obtain a remainder of 5; the fifth day of the week, Thursday.

$$\begin{array}{r l} 1912 & \text{less } (20)\ 19+1 \\ \hline 1892 & \\ 478 & \\ 53 & \\ 4 & \\ \hline 7)\overline{2427} & \\ \hline 346 \ldots 5, & \text{Thursday.} \end{array}$$

To find the Day of the Month on which a Particular Day of the Week will happen.

EXAM. 2. The presidential election occurs on Tuesday after the first Monday in November; *what day of the Month* will it be in 1912?

To solve a problem of this kind, we have to find, first, on what day of the week November 1 will happen.

SOLUTION.— 1912 being a leap year we subtract 20 (19 + 1) from the given year, as in the foregoing example, next, adding to the remainder one-fourth of 1912, also the number of days, 306, from January 1, up to and including November 1; and 4 (19 ÷ 4) and dividing by 7, we get a remainder of 6; the sixth day of the week, Friday, on which November 1 will happen; consequently election day will be the following Tuesday, or Nov. 5.

$$\begin{array}{r l} 1912 & \text{less } 20 \\ \hline 1892 & \\ 478 & \\ 306 & \\ 4 & \\ \hline 7)\overline{2680} & \\ \hline 382 \ldots 6 & \text{Friday.} \end{array}$$

EXAM. 3. Abraham Lincoln was born in Kentucky, February 12th, 1809, on what day of the week did it happen?

SOLUTION.— Deducting 18 from the given year leaves 1781; then proceeding according to the rule we find the remainder to be 1; the first day of the week, Sunday.

$$\begin{array}{r} 1809 - 18 = 1781 \\ 452 \\ 43 \\ 4 \\ \hline 7)\overline{2290} \\ \hline 327 \ldots 1 \end{array}$$

On what day will it happen in 1912? Ans. Monday.

EXAM. 3. If your birthday be Dec. 29, 1900, on what day of the week will it happen?

$$\begin{array}{r} 1900 - 19 \\ \hline 1881 \\ 475 \\ 363 \\ 4 \\ \hline 2723 \\ \hline 389 \ldots\text{Saturday.} \end{array}$$

Subtracting 19 from 1900, we get 1881. Adding the required numbers according to the rule and dividing by 7, we find the day to be Saturday, the division being exact.

BISSEXTILE OR LEAP YEAR.

NOTE. — The solar year, or the time required by the earth to go once around the sun, is 365 da. 5 h. 48 min. 48 sec. The common year is 365 days. Hence,

1	solar year is		5 h.	48 min.	48 sec.	longer than	1	common year.
4	" years are		23 "	15 "	12 "	" "	4	" years.
100	" " "	24 da.	5 "	20 "		" "	100	" "
400	" " "	96 "	21 "	20 "		" "	400	" "

or 97 days nearly. Hence,

If 97 days be added to every 400 years, in other words, if 97 leap years be reckoned in every 400 years, the calendar will be only 2 h. 40 min. in advance of true time; or about 1 day in 4000 years. To be more explicit, let us take the time between the years 1600 and 2000, a period of 400 years. It is clear from the foregoing that we cannot reckon every fourth year in these 400 a leap year as that would give 100 leap years, while the correct number is 97, a difference of 3 years. To distribute those 97 days, then, among 97 years, every fourth year in the 400 is reckoned a leap year, except the centennial years 1700, 1800 and 1900. Hence,

I. *Every year that is exactly divisible by 4 is a leap year, the centennial years excepted; the other years are common years.*

II. *Every centennial year that is exactly divisible by 400 is a leap year; the other centennial years are common years.*

The year 1900, for example, is a common year, because, although exactly divisible by 4, it is not exactly divisible by 400. The year 1904 is a leap year, being exactly divisible by 4; and the years 1600, 2000 and 2400 are leap years, being exactly divisible by 400.

APPENDIX.

INTEREST RULES TERSELY STATED AND EXPLAINED.

GENERAL RULE.—*To find the interest of a given sum for any number of days, at any rate per cent., on the basis of* 360 *days to the year:* Multiply the principal by double the rate, and the product by the days, cut off five figures from the right, *counting from the decimal point always,* and add a third and a sixth.

EXAM. What is the interest of \$3765 for 8 days at $3\frac{3}{4}\%$?

$$\begin{array}{r|l} \multicolumn{2}{c}{\$3765} \\ \hline 2 & 25900 \\ & 753 \\ & 1255 \\ \hline \$3 & 1375 \end{array}$$

Doubling the rate, $3\frac{3}{4}$, we get $7\frac{1}{2}$; multiplying this by 8, the days, gives 60 ; then 60 times the principal is \$225900. Cut off five figures to the right, and add $\frac{1}{3}$ and $\frac{1}{6}$ of that third ; the interest is \$3.1375.

NOTE.—When the principal is not large enough to cut off five figures, *prefix* a cipher or ciphers, to make five, and proceed according to the rule. Thus, if the principal were \$65, in the example, $65 \times 60 = \$3900$; prefixing a cipher we have .03900, five decimal places, and adding a third and a sixth of that third we get .05446, the interest of \$65 for 8 days at $3\frac{3}{4}\%$. When there are cents in the principal there will, of course, be seven places of decimals.

The reason of the rule will be understood from the following : The problem fully expressed is this : If \$100 earn $\$3\frac{3}{4}$ in 360 days, what will \$3765 earn in 8 days ? And the solution, by compound proportion, would be as follows :

$$\$100 : \$3765 :: \$3\tfrac{3}{4}$$

$$360 \text{ days} : 8 \text{ days}$$

Reducing this to a simple proportion, we have the following:

$$36000 : 30120 :: 3\tfrac{3}{4}$$

Doubling the rate, now, doubles the first term of the proportion, and we have $72000 : 30120 :: 7\frac{1}{2}$.

Multiplying now by $7\frac{1}{2}$, double the rate (we have already multiplied by the days, 8), and dividing by 72000, solves the problem.

Instead of dividing by 72000, in the usual manner, we prefer to make use of a more simple divisor, and that we get by adding a third and a sixth of that third, making 100000, as shown in the margin.

$$\begin{array}{r} 72000 \\ 24000 \\ 4000 \\ \hline 100000 \end{array}$$

Bearing in mind now, that whatever operation is performed on the divisor to simplify the division, a similar operation must be performed on the dividend; hence we add to 225900 a third and a sixth of that third, having cut off five figures from the right which divides by 100000.

METHOD OF EXACT INTEREST.

GENERAL RULE.—*To find the interest of a given sum for any number of days, at any rate per cent., on the basis of 365 days to the year:* Multiply the principal by double the rate, and the product by the days, cut off five figures from the right, counting from the decimal point, and add a third, a tenth and a tenth.

EXAM. What is the interest of \$275000 for 50 days, at 2% ?

$$\begin{array}{r|l} \$275000 & \\ \hline 550 & 00000 \\ 183 & 33333 \\ 18 & 33333 \\ 1 & 83333 \\ \hline \$753 & 50000 \\ & 7530 \\ \cline{2-2} & 42 \end{array}$$

Doubling the rate, 2, gives 4; then 50 days multiplied by 4 gives 200. Multiplying the principal, now, by 200, cutting off five figures from the right, and adding a third, a tenth of that third and a tenth of that tenth, we get $753.50. This gives an excess of 10 cents in every $1000 of the interest, or 1 cent in every $100, so we bring 10 times the dollars, or 7530, to the right, and subtracting, we get the correct interest, $753.42.

The reason of the rule will be understood from the following: The problem fully expressed is this: If $2 be paid for the use of $100 for 365 days, how much will be paid for the use of $275000 for 50 days, at the same rate? And the solution by compound proportion would be as follows:

$$\begin{array}{c} \$100 : \$275000 :: \$2 \\ 365 \text{ days} : 50 \text{ days} \end{array}$$

Reducing this to a simple proportion, we have the following:

$$36500 : 13750000 :: 2$$

Doubling the rate now, doubles the first term also, and we have:

$$73000 : 13750000 :: 4$$

Multiplying the middle term now by 4, and dividing the product by 73000 solves the problem.

Instead of dividing by 73000, however, we prefer to use a more simple divisor, and that we get by adding a third, a tenth of that third and a tenth of that tenth, making 100010, as shown in the margin.

$$\begin{array}{r} 73000 \\ 24333\frac{1}{3} \\ 2433\frac{1}{3} \\ 243\frac{1}{3} \\ \hline 100010 \end{array}$$

Performing a similar operation on the dividend 55000000, we add its third, a tenth of that third and a tenth of that tenth, getting 75350000 for new dividend, to correspond with 100010, the new divisor. Cutting off five figures divides by 100000 (which, by the way, may be done at the beginning or end of the process), and rejecting 10 times the quotient, or the dollars, $753,

makes the required correction. (See example 3, page 53; also rule, page 54.)

NOTE.—If the principal be not large enough to give five decimal places, prefix a cipher, or ciphers, to make five, as in the rule for 360 days.

If we had to use 366 days (leap year) as divisor, the complete divisor would then be the double of 366, or 732, multiplied by 100, or 73200, and by adding to this a third and a tenth of that third, the new divisor would be 100004, a simple divisor. (See example 4, page 54.)

SPECIAL RULES.

Although the foregoing rules are general, there are special rules for certain rates which will be found shorter for business purposes. Foremost among these we would place the

SIX PER CENT. METHOD.

NOTE.—This method has been already fully explained in the chapter on Interest, commencing at page 122, where we have made proportion and cancellation the groundwork of the rule, but for persons who may not have a knowledge of these subjects, the following will be found, perhaps, more simple and clear:

RULE. *To find the interest of any sum for one year at any rate per cent.:* Multiply the principal by the rate per cent. and divide the product by 100.

EXAM. What is the interest of \$475 for one year at 6%?

Multiplying by 6, the rate. and dividing by 100, we get \$28.50, the interest.

$$\begin{array}{r} \$475 \\ \hline 28|50 \end{array}$$

Suppose, now, we wished to find the interest of \$475 for 2 months, or 60 days, at 6%, on the basis of 360 days to the year, we simply take $\frac{1}{6}$ of \$28.50 (2 mos. or 60 days being $\frac{1}{6}$ of a year), which gives the required interest, \$4.75, or a cent for every dollar of the principal, showing that we need never figure 2 mos., or 60 days' int. at 6%, only call the dollars of the principal so many cents, in other words, 1% of the principal is always the interest for 60 days at 6%.

Taking this for our basis, it is evident that the int. of	\$475
for 6 days at 6%, is a tenth of 60 days' interest, \$4.75, or	.475
and for 3 days, the interest is half of 6 days' interest, or	.2375

Again the interest of \$475 for 600 days, 20 mos., or 1 yr. 8 mos.

at 6%, is 10 times the int. for 2 mos. or 60 days, \$4.75, or	\$47.50
and for 40 mos., or 3 yrs., 4 mos., it is 2 times \$47.50, or	\$95.00

The rule will be found equally simple when the number of days is not a measure or a multiple of 60 or 6.

Thus, if it were required to find the int. of \$475 for 13 days at 6%.

Moving the decimal point two places to the left, gives the interest for 2 mos. or 60 days, and moving it three places, gives the interest for 6 days, at 6%, always.

Now the interest of \$475 for 6 days is	.475
And for 12 days, it is 2 times .475, or	.950
And for 1 day it is $\frac{1}{6}$ of .475, or......	79
Giving the interest for 13 days or	\$1.029

And the interest of \$475 for 117 days, at 6%, would be found thus:

The interest for 60 days is	\$4	75
And for 120 days, it is 2 times \$4.75, or.......... ...	9	50
For 3 days, the int. is half of .475, or.................		2375
Which is deducted, leaving the int. for 117 days or....	\$9	2625

EXAM. What is the interest of \$420 for 279 days, at 6% ?

Dividing 279 days by 60, it is contained 4 times, with a remainder of 39. (See note 3, page 142.)

The interest of \$420 for 60 days is	\$4	20
And 4 times \$4.20 is the int. for 240 days, or	16	80
The interest for 30 days is half 60 days' int. or	2	10
And 3 times 21 cts. (the int. for 3 days, or $\frac{1}{10}$ of 30) is 9 d. or		63
Giving the interest of \$420 for 279 days...............	\$19	53

Now since the interest of \$420 for 279 days is the same as the interest of \$279 for 420 days, the foregoing example can be simplified by taking the dollars for the days, thus:

The interest of $279 for 60 days, at 6%, is..............	$2	79
And for 420 days, it is 7 times 60 days' interest, or......	$19	53

(This method can be always adopted when it is more convenient to take the dollars for the days.

NOTE.—It need scarcely be remarked that should the principal contain cents, it will not affect the process, the figures to the right of the line being simply decimals.

EXAM. What is the interest of $345.60 for 1 y. 10 mos. 26 d., at $4\frac{1}{2}\%$?

Moving the decimal point one place to the left, gives the interest for 20 months, or for 1 y. 8 mos., at 6%.......	$34	560
The interest for 2 mos. is a tenth of that, or...........	3	456
And for 20 days, the interest is a third of 2 mos., or....	1	152
The interest for six days is..........................		345
Giving the interest of $345.60 for the given time, at 6%,	39	513
Deducting $\frac{1}{4}$ of the interest at 6%, $39.513, or..........	9	878
Gives the interest at $4\frac{1}{2}\%$, or..........................	$29	635

NOTE.—The difference between 6% and $4\frac{1}{2}\%$ is $1\frac{1}{2}$; and $1\frac{1}{2}$ is $\frac{1}{4}$ of 6. (See page 124, also rule, page 125.)

INTEREST ON RUNNING ACCOUNTS.

Instead of finding the interest on each item separately, as is frequently the case with persons having to deal with such matters, it will be found much more simple and expeditious to proceed as follows:

EXAM. What is the interest of $3000 for 23 days; $4500 for 10 days; $5000 for 19 days; $2000 for 26 days; and $4000 for 17 days, at 6%?

Multiply each principal by the number of days respectively; cut off three figures to the right, from the sum of the products; divide by 6, and the result is the interest at 6%.

Process.

$$
\begin{array}{l}
\$3000 \times 23 = \quad 69000 \\
\$4500 \times 10 = \quad 45000 \\
\$5000 \times 19 = \quad 95000 \\
\$2000 \times 26 = \quad 52000 \\
\$4000 \times 17 = \quad \underline{68000} \\
\qquad\qquad\qquad\quad 329|000 \\
\qquad\qquad\qquad\quad \$54|833
\end{array}
$$

NOTE.—For any rate other than 6%, add or subtract the difference, as pointed out at page 124.

EXAM. Find the interest on the following at 6% :

Process.

$2700 for 3 y. 5 mos. 12 days.	8100	13500	32400
$1500 for 5 y. 7 mos. 8 days.	7500	10500	12000
$4500 for 7 y. 9 mos. 13 days.	31500	40500	58500
$3800 for 1 y. 3 mos. 20 days.	3800	11400	76000
$5000 for 2 y. 0 mo. 28 days.	10000	....	140000
$7500 for 4 y. 1 mo. 0 days.	30000	7500	
	909.00	834.00	318.900
	$5454.00	$417.00	$53.15

Adding the three results thus found, we get the required interest, $5924.15.

RULE I. *Multiply the principal by the years, months and days, setting the product of each respectively in a separate column, as shown in the example, and add.*

II. *Point off two figures to the right in the sum of the yearly column, multiply by the rate, and the result is the interest for the years at the given rate.*

III. *Point off two figures in the monthly column and take half; and for the daily column point off three figures and divide by* 6 ; *the result in both cases will be the interest at* 6% *from which the interest at any other rate is readily found by addition or subtraction, as already pointed out.*

NOTE.—*Reason of the rule:* The interest of $2700 for 3 years is the same as the interest of $8100 for 1 year; the interest of $2700 for 5 months, the same as $13500 for 1 mo.; and of $2700 for 12 days the same as $32400 for 1 day, at any rate per cent.; consequently the interest of the totals, $90900; $83400 and $318900, for 1 year, 1 mo. and 1 day respectively, is equal to the interest of the several principals for the given time and rate.

Pointing off two figures from the right of the dollars in any principal, and multiplying by the rate, gives the interest of that principal for 1 year at the given rate.

Pointing off two figures from the right of the dollars in any principal, gives the interest for 2 mos. at 6%, and half of this is one month's interest.

Finally, pointing off three figures gives the interest for 6 days at 6%, and therefore the interest for 1 day is 1-6 of that.

IMPORTANT FACTS TO BE REMEMBERED.

(1.) That the interest of any principal for 6000 days, at 6%, is equal to the principal itself; in other words, any sum of money will double itself at 6%, simple interest, in 6000 days, 200 months, or 16⅔ years.

(2.) Moving the decimal point one place to the left in any principal, gives the interest of that principal for 600 days, at 6%.

(3.) Moving the point two places gives the interest for 60 days ; and

(4.) Moving it three places gives the interest for 6 days, at 6% : thus :

The interest of	$4765	for	6000 days,	at 6%, is	$4765
" "	$4765	"	600 days,	"	$476.5, or $476.50
" "	$4765	"	60 days,	"	$47.65
" "	$4765	"	6 days,	"	$4.765

And from this simple basis the interest for any time and rate can be easily found, and often by a choice of two methods : (See exam. page 181).

EXAM. Find the interest of $2000 for 119 days, at 6%.

First Method.

$20	00= 60 days' interest.		
40	00=120 " "		
	33= 1 " "	1-6 of $2, or of 6d. int.=33c.	
$39	67=119 " "		

Second Method

Take the days for the dollars, and the dollars for the days :

Int. of $119 for 6000 days is $119.

" " " 2000 " =⅓, or $39.67

A SIMPLE METHOD FOR AVERAGING ACCOUNTS.

In averaging, there are two kinds of equations, Simple and Compound.

A Simple Equation has reference to one side of an account only, which may be either a debit or credit.

A Compound Equation has reference to both sides of an account.

Simple Equation.

If one person owe another, on Jan. 1, \$300 payable in 4 months, \$500 payable in 6 months, and \$400 payable in $10\frac{1}{2}$ months; at what time may the whole be paid without loss to either party?

Process.

$$
\begin{array}{rcrcl}
\$300 & \times & 4 & = & 1200 \\
500 & \times & 6 & = & 3000 \\
400 & \times & 10\frac{1}{2} & = & 4200 \\
\hline
\$1200 &) & & & 8400(7 \\
 & & & & 8400
\end{array}
$$

The interest of \$300 for 4 months equals the interest of \$1200 for 1 mo.; the interest of \$500 for 6 mos. equals the interest of \$3000 for 1 mo.; and that of \$400 for $10\frac{1}{2}$ mos. equals the interest of \$4200 for 1 mo. And since the interest of \$8400 for 1 month equals the interest of \$1 for 8400 months, \$1200 = 7 months.

The time, therefore, is 7 months from Jan. 1, or Aug. 1.

RULE. *Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments; the quotient will be the time to be counted forward from the date at which the credits begin.*

EXAM. What is the equated time of payment for the following bill?

NEW YORK, *Jan.* 1, 1893.

EDWARD JONES.

To JAMES FRENCH *Dr.*

1892.	June 5,	To Cash	....	$300
"	July 20,	" Mdse., 3 mos	....	400
"	Aug. 10,	" Mdse., 2 mos	....	200
"	Oct. 1,	" Cash	....	500

Instead of proceeding school-boy fashion, arranging due dates, counting the days, etc., let us treat the time as months and the fractions of a month, as in the preceding example.

NOTE.—It need scarcely be remarked that, in averaging an account, we can assume the first date on the bill, or the last date, or any date outside the bill, as the date from which to reckon, or the *focal date* as it is called.

Mo.						
0	June 5,	$300	1500		400	
1	July 20, 3 mos.	400	8000		1600	
2	Aug. 10, 2 mos.	200	2000		800	
4	Oct. 1,	500	500	1200|0	2000	
		$1400)			4800	(3 mos.
					4200	
					600	
					18000	(13 days.
					18200	

Sept. 13*th.*

Explanation.— Assuming for convenience, the last day of the month previous to the earliest date of the bill, or May 31, as the *focal date*, we see that the time on the first item is 5 days; on the second, 4 mos. 20 days; on the third, 4 mos. 10 days, and on the last item it is 4 mos. and 1 day.

The days or dates are fractions of a month, viz., $\frac{5}{30}$, $\frac{20}{30}$, $\frac{10}{30}$, etc., and instead of multiplying by each fraction separately, we simply multiply by the numerators, or dates, add the products and divide by 30 (cut off one figure and divide by 3), which gives a monthly product.

Dividing 12000 by 30, we get 400 which is carried to the right, as shown in the margin; next, we multiply by the months, adding those on the left-hand margin to these on the right of the dates, viz., 1 and 3, or 4; then 4 times $400, or 1600, is set under 400, to the right; now 4 (2 + 2) times 200, or 800, and finally 4 times 500, or 2000. Then dividing the sum of the products, 4800, by the sum of the payments, 1400, we get 3 mos. and a remainder of 600.

Multiplying this rem. by 30, and dividing the product, 18000, by 1400, we get 13 days, nearly. The time, then, is 3 mos. and 13 days, counted forward from May 31, giving the average due date Sept. 13th.

Note.—The advantage of arranging the time in months on the margin, as shown in the example, will be apparent to the reader, as it enables us to see at a glance the whole time on each bill, including the *term of credit.*

To make this simple method of average more clear, it may be well to remark that, had we multiplied $300 by $\frac{5}{30}$ mos.; $400 by $4\frac{20}{30}$; $200 by $4\frac{10}{30}$ and $500 by $4\frac{1}{30}$, and added the results, we would have got $4800, as found by the short process on opposite page.

Again, since $300 for 5 days = $1500 for 1 day; $400 for 20 days = $8000 for 1 day, &c.; therefore, $12000 for 1 day equals the several bills for their respective number of days. Dividing $12000, the amount for 1 day, by 30 (days in a mo.), gives $400 for 1 mo., or the result for all the dates or fractions of a month.

Multiplying each bill, now, by its respective number of months, setting the results under $400 and adding, we get $4800, or the result for 1 month. Now, if we have the use of $4800 for 1 mo., how long ought we to have the use of $1400, the amount of bill? Ans. As often as $1400 is contained in $4800; the statement by proportion being; As $1400 : $4800 : : 1 mo. Dividing 4800, now, by 1400, we get 3 mos. and 13 days, which means that the interest of $4800 for 1 mo. = the interest of $1400 for 3 mos. and 13 days.

Compound Equation.

Dr. JOHN SMITH. Cr.

1893.					1893.				
Jan.	12	To Mdse., 2 mos.	100	00	Mar.	10	By dft. 60 days	150	00
Mar.	12	" " 1 mo.	300	00	June	26	" " 90 days	100	00
July	20	" " 3 mos.	200	00	Oct.	15	" Cash	100	00

Find the equated time of payment, and the cash balance of the foregoing account, Jan. 1st, 1894, int. at 6%.

Process.

Mo.			Dr.		293
0	Jan. 12	2 mos.	100	1200	200
2	Mar. 12	1 mo.	300	3600	900
6	July 20	3 mos.	200	4000	1800
			600	880|0	3193

Mo.			Cr.		212
2	Mar. 10+3	2 mos.	150	1950	600
5	June 26+3	3 mos.	100	2900	800
9	Oct. 15		100	1500	900
			350	635|0	2512
			250)		681(2 mos.
					500
					181
					5430(22 d.
					5500

Ans. March 22; Cash balance, $261.63.

Arranging the time on both the debit and credit sides of the account as pointed out in the preceding example, we find the sum of the payments on the Dr. side 600, and the sum of the products, 3193.

And on the Cr. side the sum of the payments is 350, and the sum of the products, 2512.

Taking the two latter sums from the former, we find the balance of acct. 250, and the difference of the products, 681. Dividing the latter by the former, we get the time 2 mos. 22 d., and counting forward we find the average date to be March 22d.

Now the time from March 22, 1893, to Jan. 1, 1894, is 9 months and 9 days, and the interest of $250, the balance of acct. for that time, at 6%, is $11.63, making the balance due, $261.63.

NOTE 1. If the balance of the acct. be not contained in the difference of the products, multiply the latter by 30, then divide and the result will be days.

NOTE 2. It may be well to remark that, in case of a note or draft, the days of grace are to be taken into consideration, as in the two first items of the credit side where we have multiplied by 13 and 29 days.

There may be such a combination of debts and credits, that the equated time will be earlier or later than any date of the account, as in the following example.

DR. JOHN BROWN. CR.

1894					1894				
May	4	To Mdse	500	00	July	9	By Cash	400	00
June	16	" "	800	00	Sept.	11	" Mdse	600	00
July	21	" "	300	00	Oct.	29	" Cash	200	00

Process.

DR.

mo.				
0	May 4 × $500 =	2000	703
1	June 16 × 800 =	12800	800
2	July 21 × 300 =	6300	600
	$1600	2110|0	2103
	1200		
	$400		

CR.

mo.			
			533
2	July 9 × $400 =	3600	800
4	Sept. 11 × 600 =	6600	2400
5	Oct. 29 × 200 =	5800	1000
	$1200	1600|0	4733
			2103
			2630

$2630 \div 400 = 6$ mos. 17 d.

To be counted *backward* from the focal date, April 30, making the average date Oct. 12, 1893.

Here, we observe, that although there is a balance of $400 on the Dr. side, the Cr. side has an excess of $2630 for 1 month; in other words, J. B. has had the advantage of that amount for 1 month more than his creditor; hence, he should pay the balance, $400, soon enough to make the terms of credit equal.

Now, since $1 = 2630 mo.

$400 = 6 mos. 17 days;

that is, the interest of $1 for 2630 months, or $2630 for 1 month equals the interest of $400 for 6 mos. 17 d.

Hence the following

RULE.— I. *Arrange the time in months, on the margin, taking the last day of the month previous to the earliest date of the account, as the focal date.*

II. *Multiply each payment by its respective date, add the several products, cut off one figure, and divide by* 3.

III. *Under the result, set the products of the payments by the months, and take the difference between the debit and credit products.*

IV. *Divide this difference by the balance of account; the quotient will be the average term of credit, to be reckoned* FORWARD *from the focal date when the balance of products and balance of account are on the same side of the account; but* BACKWARD *when the balances are on opposite sides.*

V. *If the balance of account becomes due before the date of settlement, add the interest for the interval, at the legal rate; if after, deduct the interest.*

MONTHLY STATEMENT.

NOTE.—It is scarcely necessary to remark that, in averaging an account, the cents in the payments are not multiplied, the rule being to reject them when less than 50, and when 50 cents, or more, to add $1.

EXAMPLE.

1894.	Jan. 2.	To Mdse..	$385	770		8
	" 4.	" " .	270	1080		12
	" 6.	" " ..	420	2520		24
	" 7.	" " ..	560	3920		42
	" 10.	" " ..	345	3450		30
	" 12.	" " ..	412	4944		48
	" 15.	" " ..	520	7800		75
	" 18.	" " ..	600	10800		108
	" 20.	" " ..	480	9600		100
	" 25.	" " ..	160	4000		50
	" 28.	" " ..	40	1120		00
	" 30.	" " ..	180	5400		60
	" 31.	" " ..	200	6200		62
			$4572)	61604(13		46)619(13

Multiplying each payment by its respective date, and adding, we find the sum of the payments to be $4572, and that of the products, 61604. Dividing the latter by the former, we get Jan. 13th the average date.

The same result, observe, is obtained by simply multiplying the dollars by the date, omitting the two right-hand figures of the dollars, taking what is nearest the true result always, thus: Call $385, 4 and multiply by 2 (that is, $\$400 \times 2$), $270, $300, or 3×4; 4×6, etc., and divide by 4600, that is, 46.

NOTE.—Count from, and include, the first day of the month always.

Average the following account. Terms, 60 days: —

Mo	1895.					
0	April 3.	To mdse.	$16		48	
	" 7.		25		175	
	" 15.		8		120	39
1	May 6.		12		72	
	" 7.		10	$34	70	
	" 12.		5		60	34
	" 25.		7		175	
3	July 8.		15		120	
	" 12.		20	41	240	123
	" 16.		6		96	
			$124		3|0)117|6	124)196(1 mo.
					39	124
						72
						30
						124)2160(17
						2108

Arranging the time in months on the margin, we find 0 mo. for April, 1 mo for May and 3 mos. for July

Multiplying each item of the bill now by its respective date, and adding the results, we get $1176 for 1 day. Dividing this by 30 (days in a mo.) gives $39 for 1 mo., which is carried to the right, opposite the last date of April. Then $34, the amount for May, is multiplied by 1 mo. and $41, for July, is multiplied by 3 and the results set under $39 and added, making $196 for 1 mo. Dividing 196 now by $124, the amount of bill, we get 1 mo. and 17 days. Counting 1 mo. and 17 days forward from April 1, inclusive, we get May 17th for the average date; and counting 60 days (term of credit), forward from May 17, gives July 16 for the due date. (*See note, page* 187.

To impress more thoroughly upon the mind of the student, a knowledge of this important subject, it may be well to give the solution of one more example, as follows:

John King wishes to settle his account on Dec. 1, 1895; how much does he owe, charging interest at the rate of 6%?

1895. **Dr.**

Mo.							
0	May	7.	To mdse.	$25		175	
	"	9.	" "	30		270	
	"	15.	" "	20		300	82
2	July	1.	" "	5		5	
	"	7.	" "	40	$105	280	210
	"	12.	" "	60		720	
5	Oct.	3.	" "	8		24	
	"	10.	" "	52	$72	520	360
	"	15.	" "	12		180	
				$252		3|0)247|4	$652
						82	

Cr.

1	June	12.	By Cash.	$50	$70	600	45
	"	15.	" "	20		300	70
5	Oct.	1.	" "	100	$130	100	650
	"	12.	" "	30		360	
				$200		3|0)136|0	$765
				$52		45	52)113(2 mo.
							104
							9
							30
							52)270(5 days.
							260

Solution.—Counting May 0, July 2, Oct. 5 on the Dr. side, and June 1 and Oct. 5 on the Cr., we have the time arranged at sight on the margin. Proceeding now according to the rule, we get $652 for 1 mo. on the Dr. and $765 on the Cr. side. The balance of account is $52, and is on the Dr. side; the balance of interest is $113, and is on the Cr. side. In other words, the balances are on the opposite sides of the account, and therefore the time, 2 mos. and 5 days (found by dividing the balance of interest by the balance of account), is to be reckoned *backward* from May 1, giving Feb. 23d for the average date. Interest is now charged on $52 from Feb. 23d to Dec, 1 (date of settlement), at 6%. The time is 9 mos. and 6 days; the interest, $2.39, making the balance due, $54.39.

SHORT METHODS.

Exam. What is the cost of 1278456 pounds of iron at $24.81\frac{3}{4}$ per gross ton (2240 lbs.)?

Long Method.

```
          1278456
           24.81¾
        ---------
           639228
           319614
           278456
         10227648
         5113824
        2556912
        ---------
2240)31728081.78($14164.322
     2240
      9328
      8960
       3680
       2240
       14408
       13440
         9681
         8960
          7217
          6720
           4978
           4480
            4980
            4480
```

Short Method.

127845	6
18263	657
9131	828
4565	914
456	591
5	707
2	853
1	426
$14164	322

Dividing the number of pounds by 70, gives the price at $32.00

$\frac{1}{2}$ of $32	"	"	16.00
$\frac{1}{2}$ of $16	"	"	8.00
$\frac{1}{10}$ of $8	"	"	.80
$\frac{1}{80}$ of this	"	"	.01
$\frac{1}{2}$ of this	"	"	$\frac{1}{2}$
$\frac{1}{2}$ of this	"	"	$\frac{1}{4}$

Adding we get price at $24.81\frac{3}{4}$

(See exam. 1, page 109.)

Note.—The division by 2240 can always be simplified by cutting off one figure from the right of the dividend, counting from the decimal point, and taking a quarter, one-seventh, and one eighth. thus:

3172808	178
793202	044
113314	577
$14164	322

Cutting off one figure and dividing by 4, divides by 40; then one-seventh of that quarter, and one-eighth of that seventh, because these numbers are factors of 2240 ($40 \times 7 \times 8 = 2240$).

EXAM. What is the cost of 259356 pounds of iron at $13.75 per gross ton?

25935	6
3705	085
926	271
463	136
115	784
57	892
28	946
$1592	029

Cutting off one figure and dividing by 7, gives the

price at	$32.
One-fourth of $32 is the price at	8.
One-half of $8 " "	4.
One-fourth of $4 " "	1.
One-half of $1 " "	.50
And one-half of this "	.25
The total sum is the price at..........................	$13.75

Or thus:

Cutting off one figure, and dividing by 8, divides by 80, and gives the price at $28 per gross ton always.

25935	6	
3241	95	
1620	975	..$231.568
28	946	
$1592	029	

Half of $28 is the price at $14, or 25c. more than the given price. To get the price at 25c. we set one-seventh of the price at $14, or $231.568, a little to the right of the work, as shown in the margin; this is the price at $2, and an eighth of this, or $28.946, is the price at 25c. Subtracting this from the price at $14, gives the price at $13.75.

NOTE.—From the foregoing examples, it will be readily seen how easily the cost of any number of pounds, at $14, 16, 21, 24, 25, 28, 32, 34, 36, or in fact any price per gross ton, may be found. (See note to exam. 3, page 111.)

EXAM. What is the cost of 562800 pounds of iron at $37.50; of coal, at $3.75; and freight at 37½c. per gross ton?

IRON.			COAL.			FREIGHT.		
56280	0		5628	00		$562	800	
8040	00 =	$32.00	804	00 =	$3.20	80	400 =	.32
1005	00 =	4.	100	50 =	.40	10	050 =	4
251	25 =	1.	25	125 =	.10	2	512 =	1
125	625 =	.50	12	562 =	5	1	256 =	½
$9421	875 =	$37.50	$942	187 =	$3.75	$94	218 =	.37½

Cutting off one figure and dividing by 7, gives the price at $32 per gross ton always; cutting off two, and dividing by 7, gives the price at $3.20, or a tenth of $32; and cutting off three figures, and dividing by 7, gives the price at 32 cents, or the one-hundredth part of $32; and the aliquot parts are the same.

Or, we could have proceeded in either case as if the price were $37.50 per ton, and take one-tenth or one-hundredth at the finish.

OATS.

EXAM. What is the cost of 4760 pounds of oats at $59\frac{3}{16}$c. per bushel (32 lbs.)?

47	60 =	.32c.
23	80 =	.16
11	90 =	8
2	975 =	2
1	487 =	1
	185 =	$\frac{2}{16}$
	93 =	$\frac{1}{16}$
$88	04 =	$.59\frac{3}{16}$

Assuming the price at 1c. per pound, we have the price at once, at 32c. per bushel; then 16c. equals $\frac{1}{2}$ of 32; 8c. half of 16; 2c. a quarter of 8; 1 a half of 2; $\frac{2}{16}$ or its equal, $\frac{1}{8}$, is an eighth of 1c.; and $\frac{1}{16}$ is half of that.

EXAM. What is the cost of 300 bags of oats, 79 pounds net to the bag, at $25\frac{7}{8}$c. per bushel?

$$\begin{array}{r|lcr}
\$237 & 00 & = & 32c. \\
\hline
118 & 50 & = & 16 \\
59 & 25 & = & 8 \\
7 & 406 & = & 1 \\
3 & 703 & = & \frac{4}{8} \\
1 & 851 & = & \frac{2}{8} \\
 & 925 & = & \frac{1}{8} \\
\hline
\$191 & 635 & = & 25\frac{7}{8}
\end{array}$$

300 multiplied by 79, gives the number of pounds, 23700, and at 32c. per bushel, the price is \$237.00. Half that is the price at 16c., or \$118.50; at 8c. the price is half that, or \$59.25, and 1c. is one-eighth of that, or \$7.406; $\frac{4}{8}$, or its equal, $\frac{1}{2}$, is half of 1c., or \$3.703; $\frac{2}{8}$ is half that, or \$1.851, and $\frac{1}{8}$ is half that, or .925, making \$191.635, the price at $25\frac{7}{8}$c.

CORN.

Wheat, buckwheat, barley, etc., may be treated in like manner, by assuming the price at a cent per pound, or at as many cents per bushel as there are pounds to the bushel.

EXAM. What is the cost of 2940 pounds of corn at 67c. per bushel (56 lbs.)?

$$\begin{array}{r|lcr}
\$29 & 40 & = & 56c. \\
4 & 20 & = & 8 \\
1 & 05 & = & 2 \\
 & 53 & = & 1 \\
\hline
\$35 & 18 & = & 67c.
\end{array}$$

Cutting off two figures we have the price at 56c. per bushel; one-seventh of which is the price at 8c.; 2c. is a quarter of 8; and 1c. is half of 2, giving $35.18, the price at 67c.

And if the quantity should be given in bushels and pounds, the same method of solution can be employed, as illustrated in the following:

EXAM. What is the cost of 364 bu. 27 lbs. of oats at $42\frac{3}{8}$c. per bushel?

364	27		
29	12		
116	75	=	32c.
29	187	=	8
7	296	=	2
	912	=	$\frac{2}{8}$
	456	=	$\frac{1}{8}$
$154	601		$42\frac{3}{8}$

Multiplying the bushels by 8, and setting the result two places to the right, we get 2912; multiplying this, in turn, by 4, and adding at the same time the 27 lbs., we get 11675 lbs., or the number in $364\frac{27}{32}$ bu.

The remainder of the process needs no further explanation.

NOTE.—To reduce pounds to bushels, we should never make use of long division when the number of pounds to the bushel is a composite number. Thus, how many bushels in 11675 lbs. of oats?

11675
2918....3
364....27

Simply use the factors, 4 and 8 ($4 \times 8 = 32$). Dividing first by 4, we get 2918, and a remainder of 3. Next, we divide by 8, and we get 364 bu. and a rem. of 6; this rem. is multiplied by the first divisor, 4, and the rem. 3 added, making 27 lbs.

How many bushels in 14763 lbs. of corn ?

$$\begin{array}{r} 14763 \\ \hline 2109 \\ \hline 263....35 \end{array}$$

Here the factors are 7 and 8 ($7 \times 8 = 56$). Dividing by 7, we get 2109; dividing this by 8, we get 263 bu. and a rem. of 5, which is multiplied by 7, giving 35 lbs., or the complete rem.

HAY.

EXAM. What is the cost of 1860 pounds of hay at \$17.50 per ton (2000 lbs.)?

$$\begin{array}{r|l} 18 & 60 \\ 2 & 325 \\ \hline \$16 & 275 \end{array}$$

At 1c. per pound it is \$18.60, which is \$20 per ton. The difference between \$17.50, the given price, and \$20 is \$2.50 which is $\frac{1}{8}$ of \$20. Deducting $\frac{1}{8}$ we have the price at \$17.50.

POUNDS TO GROSS TONS.

To reduce pounds to gross tons we have the following simple Rule: Cut off one figure from the right of the pounds, and take a quarter, a seventh and an eighth; the result will be gross tons and the decimal of a ton.

EXAM. How many gross tons in 79952 lbs. of iron?

SOL.—Cutting off one figure, and taking a quarter, $\frac{1}{7}$ of that quarter, and $\frac{1}{8}$ of that seventh, we have 35.6928+ gross tons. (*See exam. page* 196.

$$\begin{array}{r|l} 7995 & 2 \\ 1998 & 8 \\ 285 & 5428 \\ \hline 35 & 6928 \end{array}$$

STERLING.

Pounds, Shillings and Pence.

Rule. *To reduce shillings, pence, etc., to the decimal of a pound: Divide the pence and farthings* (having reduced the farthings to a decimal) *by* 12; *to the result thus found prefix the shillings, and divide by* 20.

Exam. 1. Reduce 18s. $2\frac{3}{4}$d. to the decimal of a pound.

Reducing $\frac{3}{4}$ to a decimal we have $2\frac{3}{4}$ =.............. 2.75
Dividing this by 12, we have........................ .22916′
Prefixing the 18s. to this result, we have .... 18.22916′
Dividing now by 20, we have ....£.9114583′

Note.—To divide by 20, move the decimal point one place to the left and take the half.

Exam. 2. Reduce 19s. $10\frac{1}{2}$d. to the decimal of a pound.

$\frac{1}{2}$ is equal .5, and $10\frac{1}{2}$ is equal......................... 10.5
Dividing this by 12 and prefixing 19, we get........... 19.875
Taking half the result, we get .... £.99375

Reverse Rule.

To find the value of the decimal of a pound sterling to the nearest farthing: (1) *Take* $\frac{1}{5}$ *of the number expressed by the first two figures of the decimal for the shillings of the result.*

(2) *Diminish the number expressed by the remainder, with the third figure of the decimal annexed, by* $\frac{1}{25}$ *of itself, what remains will be the farthings in the rest of the required value.*

EXAM. 1. What is the value of £.99375.

Taking a fifth of 99, the first two figures of the decimal, we get 19 shillings with a remainder of 4, to which 3 (the third fig.) is annexed, making 43, this is diminished by $\frac{1}{25}$, leaving 42 farthings or $10\frac{1}{2}$ d.

.99375
19s. $10\frac{1}{2}$d.

Reason: .99375 is more nearly equal .994 than .993, and .994 = .95 + .044, and the value of .95 is found by multiplying by 20 and dividing by 100, or simply $\frac{1}{5}$ of 95 = 19s. Then the value of £.044 or £$\frac{44}{1000}$ by diminishing the denominator by $\frac{1}{25}$ we have 960 (farthings in a pound sterling), and by diminishing the numerator 44 by $\frac{1}{25}$ we get 42 nearly, hence £$\frac{44}{1000}$ is nearly equal £$\frac{42}{960}$ or 42 farthings.

EXAM. 2. What is the value of £.8525.

Taking $\frac{1}{5}$ of 85 we get 17s. and the third figure is 2 farthings, that is $\frac{2}{4}$ or $\frac{1}{2}$.

17s. $0\frac{1}{2}$d.

TO DIVIDE POUNDS, SHILLINGS AND PENCE BY 100.

To divide pounds, shillings and pence by 100, in other words, to take 1% and consequently any per cent. of sterling money, we give the following simple

RULE.—*For the pounds of the quotient take the pounds of the dividend, except the last two figures, which are to be divided by 5 for shillings; from the remainder, with half the shillings annexed, reject $\frac{1}{25}$ part and regard what remains as farthings.*

EXAM. Take 1% of £8947..13s..8d.

£89.. 9s..$6\frac{1}{2}$d.

Here by cutting off 47, the last two figures of the pounds, we have £89; and $\frac{1}{5}$ of 47 is 9, the shillings required, and the remainder is 2. This remainder with 7, the half of 14s. (because 13s. 8d. is more nearly 14s.) annexed becomes 27, from which 1 is rejected (nearly its $\frac{1}{25}$), we have 26 farthings or $6\frac{1}{2}$d.; the answer is £89 9s. $6\frac{1}{2}$d.

NOTE.—In rejecting the 1-25 always take what is nearest the true result. The reason of the process will be understood from the preceding rule.

EXAM. What is 4% of £89..16s..4d.?

Dividing 89 by 5, we get 17, the shillings, and a remainder of 4 to which 8 (the half of 16) is annexed, making 48 farthings, from this 2 (nearly $\frac{1}{25}$) is rejected, leaving 46 farthings, or $11\frac{1}{2}$d. This is 1% of the given sum. Then 4 times this is £3 11s. 10d., or 4% of the given sum.	£89 16s. 4d. 17s. $11\frac{1}{2}$d. £3 11s. 10d.

Reason of the rule: Dividing £89 by 100, we get £0, leaving a remainder of £.89. If this be multiplied now by 20 (shillings in a pound), and the result divided by 100, we get 17s. But multiplying by 20 and dividing by 100 is the same as dividing by 5; so we simply take a fifth of £.89 for the 17 shillings of the answer. This leaves a remainder of £$\frac{4}{100}$, or £.04. Now 16 shillings = £$\frac{16}{20}$ or £$\frac{8}{10}$, written decimally, £.8 is still to be divided by 100; this gives £.008. We have now £.04, the remainder left in dividing 89 by 5, plus £.008, which make, when added, £.048, or, when expressed fractionally, £$\frac{48}{1000}$. Diminishing the denominator of this fraction, now, by $\frac{1}{25}$ part of itself, we have 960; and diminishing the numerator also by a similar part of itself, we have 46 nearly; hence, £$\frac{48}{1000}$ is nearly equal to £$\frac{46}{960}$, or 46 farthings, since £$\frac{1}{960}$ = 1 farthing.

STERLING REDUCED TO AMERICAN CURRENCY.

The first three figures of the decimal of a pound will be sufficient for all practical purposes in reducing Sterling to American currency; and to find those three figures, the following simple rule is given, which will be found preferable, perhaps, to that given for finding the decimal in full, at page 201.

RULE. — (1.) *Take half the number of shillings for the first figure of the decimal, if the shillings be even; and if odd, half the shillings will be the first two figures of the decimal.* (2.) *Reduce the pence and farthings, if any, to farthings, by multiplying by* 4, *and if the result consists of only one figure, set it in the third decimal place, but if of two figures, set them in the second and third places, adding* 1 *if the number of farthings is between* 12 *and* 36, *and* 2, *if between* 36 *and* 48. (3.) *Prefix the pounds to the decimal thus found, and multiply by the rate of exchange.*

EXAM. What must be paid in New York for a bill on London for £32 18s. $2\frac{3}{4}$d., the rate of exchange being 4.87\frac{1}{2}$?

SOLUTION.— The shillings being even, half the number, or .9, is the first figure of the decimal. There are 11 farthings in $2\frac{3}{4}$d., and this number forms the second and third figures of the decimal. We have now £32 18s. $2\frac{3}{4}$d. = £32.911, and this multiplied by $4.875 = $160.44.

OPERATION.

$$\begin{array}{r} 32.911 \\ 4.875 \\ \hline \$160.441125 \end{array}$$

NOTE.— The multiplication by the rate of exchange can be frequently shortened if we assume $5 as a standard rate. In that case, at $1 per pound sterling, the cost of £32 18s. 2¾d. or its equal, £32.911, would be $32.911; at $2, two times that, and at $5 it would be five times $32.911 or $164.555.

Now multiplying by 5 is the same as multiplying by 10 and taking half, so that if we move the decimal point in £32.911 one place to the right, thus, £329.11, and take the half, we get $164.55, or the value of £32.911 sterling, in American currency, at $5 per pound sterling.

And since the difference between $5 and $4.87½ is 12½ c. or ⅛ of a dollar, we get the value at $4.87½ by subtracting ⅛ of 32 911 or $\frac{1}{40}$ of 164.55 (⅛ of $1 being the same as $\frac{1}{40}$ of $5).

SHORT METHOD

$$\begin{array}{r|l} 329 & 11 \\ \hline 164 & 55 \\ 4 & 11 \\ \hline \$160 & 44 \end{array}$$

Again, since the value of £32.911 at \$1 per pound sterling, is \$32.911, the value at 10 c. would be the tenth of that at \$1, or \$3.2911, and at 1 c. the value would be a tenth of the latter, or \$.32911; at 2 c. it would be twice \$.32911; at 3½ c. three and a half times \$.32911, etc.; so that, had the rate of exchange been \$4.88½, \$4.89½, or \$4.91, we would simply add to \$160.44 (the value at \$4.87½), once \$.32911, or 33 c.; twice \$.32911, or three and a half times \$.32911; 1, 2 and 3½ being the differences between \$4.87½ and the three mentioned rates. If the rates were less than \$4.87½, we would, of course, deduct the difference.

To illustrate further this simple method let us take the following:

EXAM. What is the value of £612 17s. 9d. at \$4.86¾?

SOLUTION. — Taking half the 17s. decimally, we have .85
for the first two figures; 4 times 9d. = 36 farthings and 1 make 37
to be set in the second and third places, making 887
the three decimal figures required. We have now £612.887 to be multiplied by the rate, \$4.86¾ which gives the required value, in American currency.

SHORT METHOD.

6128	87
3064	435
76	610
3	064
1	532
\$2983	229

Moving the point in £612.887, or, which is the same in effect, drawing the vertical line one place to the right, and taking half, we have \$3064.435, the value at \$5. Then $\frac{1}{40}$ of this (simply a fourth set in proper position) is \$76.61, which being deducted from \$3064.435, would give the value at \$4.87½. But the rate is \$4.86¾, or a difference of ¾ of a cent, which is also to be deducted, ¾ = ½ and ¼; half of \$6.128 (value at 1c.) is \$3.064, and half of this is \$1.532, the value at ¼ of a cent. Taking these three results now from \$3064.435, we have \$2983.229, the value at \$4.86¾.

NOTE.—In subtracting the three items from \$3064.435, the process is performed by addition, thus 2 and 4 are 6, and 9 (setting down 9 at the bottom), are 15 (the top figure); carry 1 to 3; 4 and 6 are 10, and 1, are 11 and 2 at bottom, are 13, etc.

Or we might proceed as follows: Suppose the rate had been $\$4.88\frac{3}{8}$, what is the value of £612 17s. 9d.?

SOLUTION.—The difference between \$5 (our standard rate) and $\$4.88\frac{3}{8}$, is $11\frac{5}{8}$ cents.

£612 17s. 9d. = £612.887, and at \$1 per £1, the value	=	\$612	887
And at \$5 per pound sterling, the value	=	\$3064	435
At 10c. per £1, the value is $\frac{1}{10}$ of that at \$1......	=	61	288
And at 1c., the value is $\frac{1}{10}$ of that at 10c....	=	6	128
At $\frac{4}{8}$c., or its equal, $\frac{1}{2}$c., the value is half 1c.. ...	=	3	064
And $\frac{1}{8}$c. is a quarter of $\frac{4}{8}$, or $\frac{1}{8}$ of 1c........	=		766
		\$2993	189

These four items make up the value at $11\frac{5}{8}$c., and deducting them from \$5, or rather from its value, \$3064.435 (by the method of addition, as pointed out in the foregoing note), we get \$2993.189, the value at $\$4.88\frac{3}{8}$.

The reason for taking half the shillings, etc., to find the first three figures of the decimal, will be understood from the following:

16s. $=£\frac{16}{20}$, $\frac{80}{100}$, $\frac{8}{10}$, or .8, all representing the same value differently expressed: therefore 17s. $= £\frac{17}{20}$ or $\frac{85}{100}$ .... = .85

And 9d. reduced to farthings $=£\frac{36}{960}$; now if we add to the denominator 960, $\frac{1}{24}$ of itself, or 40, we get 1000, and by increasing the numerator 36, by a similar part of itself, we make it 37 nearly; so that $£\frac{36}{960}$ is nearly $=£\frac{37}{1000}$ ... . = .037

Consequently 17s. 9d. expressed decimally..... .... = .887

In reducing 9d. we simply say 4 times 9 are 36, and 1 are 37; and 1 is to be always added when the number of farthings is nearest to 24, or when the number is between 12 and 36, as stated in the rule.

INTEREST ON STERLING.

RULE I. *To find the interest of any sum for* 1 *year, at any rate per cent: Multiply the principal by the rate, and divide by* 100.

EXAM. 1. What is the interest of £1 sterling for 1 year at 5% per annum?

SOLUTION.— Multiplying £1 by 5, the rate, and dividing by 100, we have £$\frac{5}{100}$, or £.05, for the interest.

But £$\frac{5}{100}$ = £$\frac{1}{20}$, or 1 shilling, or its equal, 12 pence; and since the interest of £1 for 1 year, or 12 months, at 5%, is 12 pence, the interest of £1 for 1 month is one-twelfth of 12 pence or 1 penny, and consequently the interest for 2 mos. is 2 pence; 3 mos. 3 pence, etc. Hence the following:

RULE II. *To find the interest of any number of pounds sterling for a given number of months, at 5% per annum: Take the pounds as pence and multiply by the months.*

EXAM. 2. What is the interest of £42 for 7 mos. at 5%?

SOLUTION.— Calling 42 pounds 42 pence, or 3s. 6d. and multiplying by 7, the number of months, we get £1 4s. 6d. the interest.

So likewise the interest of £42.. 10s. for 8 months at 5%, is 42½ pence, or 3s. 6½d. × 8 = £1.. 8s. 4d.

And from this the interest at any other rate may be easily derived. Thus, to find the interest at 4%, simply subtract from £1 8s. 4d. one-fifth of itself, or 5s. 8d., and we have £1 2s. 8d., the interest of £42 10s. for 8 mos. at 4%. Had the rate been 6% we should have *added* one-fifth of the interest at 5%; or we might employ the following:

RULE III. *To find the interest for months at 6% per annum: Multiply the principal by half the number of months, and divide the result by* 100.

EXAM. 1. What is the interest of £439 16s. 8d. for 1 year and 8 months, at 6% per annum?

	OPERATION.
SOLUTION.— 1 y. 8 mos. = 20 mos.	£439 16s. 8d.
Multiplying £439 16s. 8d. by 10, half of 20 mos.,	10
we get £4398 6s. 8d., and dividing this by 100,	£4398 6s. 8d.
we get £43 19s. 8d., the interest required.	£43 19s. 8d.

To divide by 100, we set down £43, leaving a remainder of 98. Taking one-fifth of 98, we get 19s. and a remainder of 3. To this remainder we annex half of 6s., or 3, making 33 from which $\frac{1}{25}$ is rejected, leaving 32 farthings, or 8d., nearly. (*For short method of dividng by* 100, *see rule page* 202; *also reason of rule page* 203.)

EXAM. 2. What is the interest of £756 14s. 10d. for 5 months, at $5\frac{1}{4}\%$ per annum.

This example is solved by both the 5% and 6% rules as follows :

First Method, 5 per cent rule.

SOLUTION.—£756.. 14s. 10d.=	$756\frac{3}{4}$d. nearly
Multiplying this by 5, the number of mos. we have	$3783\frac{3}{4}$d.
Dividing this by 12 (pence in a shiliing) we have..	315s. $3\frac{3}{4}$d.
Dividing the latter by 20 (shillings in £1) we have the interest at 5%. Now there are 20 quarters, or 4ths in 5%; therefore 1 quarter, or $\frac{1}{4}\%$ is $\frac{1}{20}$ of 5%.	£15 15s. $3\frac{3}{4}$d.
Dividing £15 15s. $3\frac{3}{4}$d. by 20 we have.............	15s. $9\frac{1}{4}$d.
Which is added making the required interest.....	£16 11s. 1d.

Second Method, 6 per cent rule.

Multiplying £756 14s. 10d. by 5, we have	£3783 14s. 2d.
And dividing this result by 2, we have...........	£1891 17s. 1d.
This multiplies by $2\frac{1}{2}$, or by half of 5 mos.....	
Dividing this by 100 (short method), we have.....	£18 18s. $4\frac{1}{2}$d.
Which is the interest at 6%.	
The difference between 6% and $5\frac{1}{4}$ is $\frac{3}{4}\%$.	
There are 24 quarters in 6%; therefore $\frac{3}{4}$ is $\frac{1}{8}$ of 24 quarters, or of 6%. Taking $\frac{1}{8}$ of £18 18s. $4\frac{1}{2}$d., we get the interest at $\frac{3}{4}\%$...... ....=	£2 7s. $3\frac{1}{2}$d.
Which is taken from the interest at 6%, leaving ... the required interest.	£16 11s. 1d.

RULE IV. *To find the interest of any sum for any number of days at any rate per cent :* (1.) *Reduce the shillings and pence, if any, to a decimal by the rule given at page* 204. (2.) *Multiply the principal by double the rate, and the result by the number of days, cut off five figures from the right, counting always from the decimal point, and add a third, a tenth of that third and a tenth of that tenth.*

NOTE.—In computing interest on sterling for days, the basis is 365 days to the year.

EXAM. 1. What is the interest of £648 15s. 3d. from June 2 to Nov. 25, at 5% per annum?

SOLUTION.—There are 176 days from June 2 to Nov. 25.		
Reducing 15s. 3d. to a decimal, we have the principal = £648.762		
Multiplying this by 10, double the rate, then by 176, we get	£11	41821.12
Cutting off five figures from the point, and taking ⅓ we get	3	80607
Taking a tenth of this third, we get........		38060
And taking a tenth of this tenth, we get		3806
Adding the four results now, we get	£15	64294
		150
Rejecting 10 times 15, or 150 from .64294, we get....		64144
Finally, the value of £.64144 is 12s. 10d. nearly,		12s. 10d

making the required interest £15 12s. 10d. (*See rule, page* 201.)

EXAM. 2. What is the interest of £8000 for 75 days at 4%?

SOLUTION.—£8000 × 600 .... =	£48	00000
Multiplying the principal by 8, and by 75, or by 600	16	
(75 × 8), at once, we get £4800000. Cutting off five	1	6
places to the right and adding a third, a tenth of		16
that third and a tenth of that tenth, we get £65.76000.	£65	76000
Rejecting 10 times 65, or 650 from this, we have		650
£65.7535. Then taking one-fifth of .75, we get 15s.,		
and the third figure of the decimal now is simply		.7535
$\frac{3}{4}$d., making the interest £65.. 15s. $0\frac{3}{4}$d.		15s. $0\frac{3}{4}$d.

(*See General Rule, page* 178; *also reason of the rule, page* 179.)

OTHER SHORT METHODS.

The six per cent rule of interest already explained in this work, can be applied to the computation of other matters, such as, Wheat, Clover Seed, Potatoes, etc., where the number of pounds to the bushel is 60.

EXAM. What is the interest of $1860 for **1 y. 9m. 5d.** at 6%?

SOLUTION.—In 1 y. 9mo. 5d., there are 635 days.

The interest of $1860 for 600 days..............	=	$186	0
Half $18.60 (60 days' int.) is the int. for 30 days..	=	9	30
One-sixth of $9.30 (30 days' int.) is 5 days.......	=	1	55
Making the interest of $1860 for 635 days.......	=	$196	85

Or, *taking the dollars for the days and the days for the dollars*, we have the interest of $635 for 1860 days, thus:

SOLUTION.—The interest of $635 for 600 days.	=	$63	5
Three times 600 days' int. is the int. for 1800 days.	=	$190	5
One-tenth of the top line, or $63.50 is 60 days....	=	6	35
And the two results, when added, is the int. for 1860 days	=	$196	85

(*See example, page* 181).

Suppose now we change the foregoing problem into the following:

EXAM. What is the price of 1860 lbs. of clover seed, at $6.35 per bushel (60 lbs.)?

SOLUTION.—At 1c. per lb., or at 60c. per bushel, the price of 1860 lbs. would be 1860c. or $18.60.

And the price of 1860 lbs. at $6, or 600c. per bushel	=	$186	0
At 30c. per bushel, it is half the price at 60c. or half $18.60..................................	=	9	30
And at 5c. the price is one-sixth of 30, or of $9.30..	=	1	55
Making the price of 1860 lbs. at $6.35	=	$196	85

The same as was found by the first solution of the interest problem. *And if the problem be reversed so as to read:* 635 lbs. of clover seed at $18.60, or 1860c. per bushel, it can be solved same as the second solution of the interest problem. The student will please try it.

WHEAT.

EXAM. What is the cost of 16940 lbs. of wheat at $87\frac{1}{8}$c. per bushel (60 lbs.)?

SOLUTION.—Cutting off two figures from 16940 gives the cost

at 1c. per pound, or at 60c. per bushel... .... =	$169	40
One-third of the cost at 60c. gives the cost at at 20c. =	56	47
At 6c. the cost is a tenth of the top line, or of 60c...=	16	94
At 1c. the cost is a sixth of that at 6c..............=	2	82
And at $\frac{1}{8}$c. it is the eighth part of 1c ....=		35
Making the total cost ....=	$245	98

POTATOES.

EXAM. What is the cost of 749 lbs. of potatoes at $47\frac{1}{2}$c. per bushel (60 lbs.)?

SOL.—Cost at 60c. ...=		$7.49
" " 30c.....=		3.75
" " 15c. ...=		1.87
$\frac{1}{6}$ of 15c. " " $2\frac{1}{2}$c....=		.31
" " $47\frac{1}{2}$c ...=		$5.93

Or thus :	
Cost at 6c. per bushel. =	$.749
And 8 times 6c. = 48c.. =	$5.992
Less $\frac{1}{12}$ of .749 = $\frac{1}{2}$c. =	62
And the cost at $47\frac{1}{2}$c. =	$5.93

HINTS ON INTEREST.

Savings banks allow interest on deposits for a certain fixed term, generally 3 mos. or 6 mos., and *calculations may be simplified in many cases by multiplying the time and rate together and dividing the result by* 12, which will give the rate for the given time, thus :

EXAM. What is the interest of $872 for 3 mos. at 4% per annum?

SOLUTION.—$\frac{3 \times 4}{12} = 1\%$, and 1% of $872 = $8.72, the interest; *the process being, of course, performed mentally.*

And if the rate were $3\frac{1}{2}\%$ per annum, we would deduct one-eighth of itself from \$8.72, and the remainder is the interest for 3 mos. at $3\frac{1}{2}\%$.

EXAM. What is the interest of \$648 for 3 mos. at $3\frac{1}{2}\%$ per annum?

SOLUTION.—The interest at 4% is simply 1% of \$648....	=	\$6.48
And at $\frac{1}{2}\%$, the difference between 4% and $3\frac{1}{2}\%$, it is $\frac{1}{8}$ of 4%.	=	.81
Making the interest for 3 mos. at $3\frac{1}{2}\%$ per annum........	=	\$5.67

Again, the interest for 3 mos. at 2% per annum, is half that at 4%; at 5% it is 4% plus $\frac{1}{4}$ and at 8% it is twice 4%, etc. And the same rule will be found to hold good in many other cases.

EXAM. What is the interest of \$648 for 1 year and 5 mos. at 7% per annum?

SOLUTION.—1 y. 5 mo. = 17 mo.; then $\frac{17 \times 7}{12} = 9\frac{11}{12}\% = 10\%$ less $\frac{1}{12}$.

The interest of \$648, at 1% = \$6.48, and at 10% it is...	\$64.80
From this we deduct $\frac{1}{12}$ of 1% or of \$6.48, equal	.54
Leaving the interest for 1 y. 5 mos., at 7% per annum.	\$64.26

And so with other problems of a similar nature.

MONTHLY PAYMENTS, 6%.

When interest is to be calculated on monthly payments, we have the following simple

RULE. (1) *Add* 1 *to the number of months.* (2) *Multiply the sum by half the number of months, and the result by the payment, and divide* by 2. (3) *If the payment be in dollars, point off two decimal places, and if there are cents, point off four places.*

EXAM. Suppose a person to pay into a Building and Loan Association, \$3 a month for 15 months; what is the amount due at the end of the time at 6% interest?

SOLUTION.— $15 + 1 = 16$; then $\frac{16 \times 7\frac{1}{2} \times 3}{2}$, or $\frac{15 \times 8 \times 3}{2} =$ $1.80
And adding the principal, $3 × 15 . . . . = 45.00

We have the amount, that is, the prin. and int. . . . = $46.80
And for

WEEKLY PAYMENTS 6%.

We have the following

RULE. (1) *Add 1 to the number of weeks.* (2) *Multiply the sum by half the number of weeks, by 7 and by the weekly payment, and divide by* 6. (3) *If the payment be in dollars, point off three decimal places, if there are cents, five places.*

EXAM. Suppose a person to pay in 50c. a week for 10 weeks; what is the amount due at the end of the time at 6% interest?

SOLUTION. — $10 + 1 = 11$; then $\frac{11 \times 5 \times 7 \times .50}{6}$ . . . . = $.032
And adding the principal, 50c. × 10. . . . . = 5.00

We have the amount. $5.032

And from this the interest at any rate other than 6% can be found by aliquot parts, as has been already pointed out.

Reason of the rule: If the payments were $1 per week, and the problem solved by the usual method, we should have multiplied $1 by 10; $1 by 9; $1 by 8, and so on down to the last payment; and adding the results, we get $55, the principal for 1 week. But we see that by adding 1 to 10, and multiplying by 5, we get $55 more readily; and 7 times $55 or $385 is the principal for 1 day, at $1 payments. Now the payment is 50c.; multiplying $385 by .50, we get $192.50, the principal for 1 day at 50c. payments. And the interest of $192.50 for 1 day at 6%, is found by multiplying by 1 day, pointing off three figures from the decimal point, and dividing by 6, and we have .192 0 ÷ 6 = .03208 +, or .032, as shown in the example, for the interest.

The same line of reasoning is applicable to the monthly rule. Multiplying $3 by 15, 14, 13, 12, and so on down to the last payment, and adding the results, we get $360, the principal for 1 month. But this is more readily found by adding 1 to 15, and multiplying by 7½, or, which is the same in effect, 15 by 8, the half of 16; then by 3, and we have $360. The interest of $360 for 2 mos. at 6% is simply 1% or $3.60; and for 1 month it is half, or $1.80, hence the reason for dividing by 2.

A SIMPLE METHOD.

To find the face of a note, the proceeds being given:

RULE. (1) *Find the interest of the proceeds for the given time and rate.* (2) *Find the interest of that interest, and so on, till the interest is so small as not to affect the result.* (3) *Add the interest thus found to the proceeds, and the sum is the face of the note.*

EXAM. For what sum must a note be drawn at 3 mos. to net $2500 when discounted at 6%.

SOLUTION.—$25.00 is the interest for 2 mos. on........ $2500.00
12.50 " " 1 mo.
Making...$37.50 " " 3 mos.
The int. of $37.50 for 2 mos. = 38c.
" " 1 mo. = 19c.
Making the int. for 3 mos. = 57c.; and the whole int. = 38.07
Adding $38.07 interest to the proceeds, we get $2538.07
the face of the note.

NOTE.—The days of grace being abolished in the State of New York, are not taken into account. If grace be allowed, add the interest for the days of grace.

Proof: The interest, or bank discount, for 3 mos. at 6% on $2538.07
(the sum for which the note is drawn), is found to be 38.07
and deducting this from $2538.07, we have the net proceeds. $2500.

EXAM. 2. Having discounted a note in bank, I am credited with $1500 as the proceeds, what was the face of the note, the rate being 4% and the time 90 days?

SOLUTION.—The interest for 90 days, or 3 mos. at 4% on $1500
is 1% or $15; and on $15, the int. is 15c. making 15.15
and adding this to the proceeds, we have the face....... $1515.15

Proof: The interest of $1515.15 for 3 mos. at 4% is 1% or 15.15
and deducting this from the face, we have the proceeds.. $1500.00

From the foregoing hints, we derive the following simple rule for computing

INTEREST FOR MONTHS.

RULE. *To find the interest for months at any rate per cent: Multiply the principal by the product of the months and rate, and divide by* 12. *Or, multiply by the months, then by the rate, and take half the result, and one-sixth of that half. Or, multiply the months and rate together and divide by* 12*; the result is the rate for the given time; multiply the principal by this rate.*

EXAM. What is the interest of \$480 for 5 mos. at 5%?

SOLUTION.— $\frac{\$480 \times 25}{12} = \$10.$; Or, $\frac{\$480 \times 5 \times 5}{2 \times 6} = \$10.$

Or, 5 times $5 = 25$, divide by $12 = 2\frac{1}{12}\%$; then $\$480 \times 2\frac{1}{12} = \$10.$

Or, because \$480, the principal, is divisible by 12, we have $40 \times 25 = \$10$, the required interest.

This rule, after a little practice, is so simple that, in numerous cases it will not be necessary to use pencil or paper in computing interest, thus: What is the interest of \$72 for 8 mos. at $3\frac{1}{2}\%$?

Ans. 8 times $3\frac{1}{2}$ is 28; then 28 multiplied by 6 $(\frac{72}{12}) = \$1.68$ the interest. Or, $28 \div 12 = 2\frac{1}{3}$, and $\$72 \times 2\frac{1}{3}\% = \$1.68$; and if there be

YEARS AND MONTHS

Reduce the years to months and proceed according to the rule.

NOTE.—In conclusion, it may be well to remark that, in relation to the foregoing rule, the *rate* may be changed for the *time* and the *time* for the *rate*, and very often to great advantage; thus, the interest of any sum for 3 mos. at 4% is the same as for 4 mos at 3%; for 6 mos. at 5%, the same as for 5 mos. at 6%; for 3 mos. and 15 days at 3%, the same as for 3 mos. at $3\frac{1}{2}\%$, 3 mos. 15 days being equal to $3\frac{1}{2}$ mos.; etc.

Suppose it were required to find the interest of \$320 for 3 mos. and 15 days, at 3%. To solve this, we *reverse* the problem so as to read: \$320 for 3 mos. at $3\frac{1}{2}\%$.

Then the interest for 3 mos. at 4% on \$320, is simply 1%..	\$3.20
and at $\frac{1}{2}\%$ it is an eighth of 4%, or 40c. deducted	40
giving the interest for 3 mos. and 15 days at 3%...................... .	\$2.80

INTEREST SIMPLIFIED.

If 360 *days be divided by the rate per cent, the interest of any sum of money for the number of days thus found, at the given rate, will be equal to one per cent of the principal.*

Thus if the rate be 4½%; dividing 360 by 4½ gives 80 days; now the interest of any sum, say $1768 for 80 days, at 4½%, is found to be 1% of that sum, or $17.68; and for 8 days the interest is one-tenth of the latter, or $1.768; for 800 days, the interest is ten times $17.68, or $176.8, and for 8000 days it is ten times the last interest, or $1768. In other words, the interest of any principal for 8000 days, at 4½%, is 100 per cent, or equal to the given principal.

Hence we have at sight:

The interest of $1768 for		8000	days, at	4½%	=$1768.
"	"	800	"	"	= 176.8
"	"	80	"	"	= 17.68
"	"	8	"	"	= 1.768
And the interst for		1 day		"	= .221

And from this simple basis the interest for any number of days is easily found. An example will make this clear.

Exam. What is the interest of $3765.47 for 92 days at 4½%?

Here we have at sight,	$37.6547	=80	days' interest;
Next, we have	3.76547	= 8	"
And half of this, or	1.88273	= 4	"
Making	$43.3029	=92	"

From the foregoing illustrations and examples the following will be readily understood:

Rate.	360 days.			
4 %	90	"	hence,	9000, 900, 90, 9 and 1 day.
$4\frac{1}{2}$%	80	"	"	8000, 800, 80, 8 " 1 "
5 %	72	"	"	7200, 720, 72, 8 " 1 "
6 %	60	"	"	6000, 600, 60, 6 " 1 "
$7\frac{1}{2}$%	48	"	"	4800, 480, 48, 8 " 1 "
8 %	45	"	"	4500, 450, 45, 5 " 1 "
9 %	40	"	"	4000, 400, 40, 4 " 1 "
10 %	36	"	"	3600, 360, 36, 6 " 1 "
12 %	30	"	"	3000, 300, 30, 3 " 1 "
etc.				

The interest of any principal, at any rate, for the number of days placed under 360, *opposite the rate, will always be one per cent of the given principal.*

Suppose now it were required to find the interest of $2460 for 93 days at 5%. (93 days=72+18+3).

Here we have at sight,	$24.60 =72	days' int.
Next, we have one-fourth of this, or	6.15 =18	"
And then one-sixth of this or	1.025= 3	"
Making	$31.775=93	"

NOTE.— If the rate be $5\frac{1}{2}$%, add a tenth of the interest at 5%, $\frac{1}{2}$% being one-tenth of 5%; and if $5\frac{3}{4}$%, add for the $\frac{1}{4}$% half the interest at $\frac{1}{2}$%, etc.

To make this simple and interesting method of interest more clear, let us take another

EXAM. What is the interest of $1872 for 94 days, at $7\frac{1}{2}$%?

First, 94 days=80+12+2.

By referring to the table we see that $7\frac{1}{2}$ is contained 48 times in 360 days. We have now, at sight, the interest of $1872 for 4800 days, 480 days and 48 days, viz. $1872, $187.2=$187.20 and $18.72

respectively; and from this, by a slight mental effort, we have the interest for 24 days, 16 days, 12 days, 8 days, 6 days, 4 days, 3 days, 2 days and 1 day.

Here then, we have the interest for 480 days	=$187.20
And for 80 days, the interest is $\frac{1}{6}$ of $187.20	= $31.20
12 days' int.=$\frac{1}{4}$ of 48 days, or $\frac{1}{4}$ of $18.72=	4.68
2 " =$\frac{1}{6}$ of 12 " , or $\frac{1}{6}$ of 4.68=	.78
Making 94 days' interest, at $7\frac{1}{2}\%$	= $36.66

And if the rate were $7\frac{3}{4}$, we would add $\frac{1}{30}$ of the interest at $7\frac{1}{2}\%$: and if $7\frac{1}{4}\%$, deduct $\frac{1}{30}$; $\frac{1}{4}$ being $\frac{1}{30}$ of $7\frac{1}{2}\%$, or 30 quarters.

To divide by 30, move the decimal point one place. to the left, or suppose it moved, and divide by 3.

NOTE.—It is scarcely necessary to remark that the interest of any sum for 90 days, at 4%, equals the interest for 45 days at 8%; in other words, when the rate is doubled the days are halved, and vice versa.

If, now, we refer to the example given at page 177, under General Rule, we find that the problem there given, can be more readily solved from a knowledge of what has been given in the table.

Now the rate given in the problem referred to is $3\frac{3}{4}\%$, which is half $7\frac{1}{2}\%$, and since 48 is the number of days which will give 1% of the principal for interest at $7\frac{1}{2}\%$, the number of days for $3\frac{3}{4}$ is 96. Or 360 days$\div 3\frac{3}{4}$=96 days. Hence

The interest of $3765 for 96 days, at $3\frac{3}{4}\%$=$37.65

and 8 " $=\frac{1}{12}$ = 3.1375

IMPORTANT FACTS ILLUSTRATED BY EXAMPLES.

If we bear in mind the fact that the interest of $1 for 2 days, at any rate per cent. is the same as the interest of $2 for 1 day; $60 for 189 days, the same as $189 for 60 days ; $5000 for 1 y. 7 mo., or 570 days, the same as $570 for 5000 days, etc.; many problems of daily occurrence in business transactions can be simplified.

EXAM. What is the interest of $8000 for 5 mo., 19 da., or 169 da. at $2\frac{1}{4}\%$?

Dividing 360 days by $2\frac{1}{4}$, the rate, we get 160 days as the basis for that rate; in other words, the interest of any principal for 160 days, at $2\frac{1}{4}\%$, is one per cent. of the principal always.

The interest, therefore, of $8000 for	160 days,	at $2\frac{1}{4}\%$	=	$80.00
" "	8 "	$\frac{1}{20}$	=	4.00
	1 "	$\frac{1}{8}$	=	.50
Making the int. for 5 mo. 19 da. or	169 "		=	$84.50

But the problem can be simplified at once by *reversing* it so as to read: $169 for 8000 days, at $2\frac{1}{4}\%$, as in this case we have the interest at sight by simply taking half $169=$84.50

REASON.—Since the interest of $169 for 160 days at $2\frac{1}{4}\%$, is $1 69, the interest for 1600 days is ten times that, or $16.90, and for 16000 days it is ten times $16.90, or $169; in other words, the interest of any principal at $2\frac{1}{4}\%$, simple interest, will equal the principal in 16000 days, and for 8000 days the interest is half the principal.

Or, we might reason thus: It has been shown at page 216 that any sum of money for 8000 days, at $4\frac{1}{2}\%$, simple interest, will double itself, or the interest will equal the principal, and at $2\frac{1}{4}\%$ it is half that at $4\frac{1}{2}\%$. And from this the interest of any part or any multiple of $8000 can be easily derived.

TO CHANGE COMMERCIAL INTEREST TO EXACT INTEREST.

RULE. *Move the decimal point in the commercial interest two places to the left; under the result set a third of itself, one tenth of that third and a tenth of that tenth; add the four numbers and take the total from the commercial interest, the remainder is the exact interest.*

EXAM. What is the exact interest of $7200 for 147 days at 5%?

Reversing the problem we have $147 for 7200 days; and at 5% the commercial interest	=	$147.
Moving the point in this two places to the left, and adding a third, a tenth and a tenth, we get $2.0139, which is taken from $147, giving the exact interest $144.9861.	$1.47 49 49 49	 $2.0139
		$144.9861

REASON.—Commercial interest exceeds exact interest by $\frac{5}{365}$, or $\frac{1}{73}$ part, and the foregoing is a simple method for dividing by 73. (*See exam.* 1, *page* 51, *and rule page* 52.)

TO CHANGE EXACT INTEREST TO COMMERCIAL INTEREST.

RULE. *Move the decimal point in the exact interest two places to the left; under the result set one-third of itself and a sixth of that third; add the sum of the three numbers to the exact interest and we have the commercial interest.*

EXAM. What is the commercial interest of a sum of money whose exact interest is $144.99?

Moving the decimal point in $144.99 two places to the left, and adding to this a third of itself and a sixth of that third, we get $2.0137, which is added to the exact interest $144.99 giving the commercial interest $147, very nearly.	$1.4499 4833 805 $2.0137 144.99 $147.0037

REASON.—Exact interest is $\frac{5}{360}$, or $\frac{1}{72}$ part less than commercial interest, and to divide by 72 we add to it its third and a sixth of that third, which gives 100 for a simple divisor, as shown in the margin, making the same additions to $1.4499 to equalize; and moving the point two places to the left divides by 100.

72
24
4
100

INTEREST ON DAILY BALANCES.

RULE. *Multiply each daily balance by the number of days it remains unchanged, and add the several products; the result will represent the principal for* 1 *day. Then if* 365 *days be taken as the basis, apply the General Rule given at page* 178; *and if* 360 *days, apply the rule given at page* 177.

EXAM. What is the interest of $136800 for 1 day at 2% (365 days)?

	$136800
Multiplying by 4, double the rate; cutting off five figures,	5.47200
and adding a third, one tenth of that third, and one tenth	1.82400
of that tenth, we have the required interest $7.49.	18240
(See rule, and reason, page 178.)	1824
	$7.49664

Balance.	Deposit.	Check.	Balance.
$8396.16	$738.31	5 0 7 . 4 8 $492.52	$8641.95

RULE. *To find the final balance: Add the complement of the check figures to the deposit and original balance. Drop* 1 *immediately to the left of the last, or left-hand figure of the check, always.*

The complement of a number is what it wants of being a unit of the next higher order. Thus, 3 is the complement of 7 (7+3=10) and 48 is that of 52 (52+48=100). The complement of a number is readily found by setting down, first, what the unit figure of that number wants of being 10; and next, what each succeeding figure wants of being 9. Thus, in the check number above, 8 is set down above the unit figure 2, so as to make 10; and next, 4, 7, 0, 5, so as to make 9's with the remaining figures. The complement of $492.52, then, is $507.48, the sum of both numbers making $1000.

Instead of subtracting $492.52, now, to find the final balance $8641.95, we add its complement $507.48. The sum of the three numbers is $9641.95; but dropping 1 immediately to the left of the check figure 4, the true result is $8641.95.

Reason of dropping 1: Subtracting a number from a unit of the next higher order, is the same as adding its complement and dropping that unit.

Thus, taking $492.52 from $1000, is the same as adding $507.48, and dropping the 1 in 1000.

$1000	$1000
492.52	507.48
$507.48	(1)507.48

NOTE.—In practice, the complement is not set down, the work being performed mentally. Thus, From the sum of $8396.16 and $738.31; take $492.52.

$8396.16 + $738.31 − $492.52 = $8641.95

Here, we say 8 and 1 are 9, and 6 are 15; 5 and carry 1; 1 and 4 are 5, and 3 are 8, and 1 are 9; 7 and 8 are 15, and 6 are 21; 1 and carry 2; 2 and 3 are 5, and 9 are 14; 4 and carry 1; 1 and 5 are 6, and 7 are 13, and 3 are 16; 6 and carry 1; 1 and 8 are 9; but here, 1 is dropped immediately to the left of 4, *in the subtractive number*, and, therefore, 8 is set down; the result is $8641.95.

(*For a fuller explanation on this matter, see page* 283.)

In computing the cost of pig iron, 28 pounds are usually allowed for sand, making 2268 pounds to the gross ton, instead of 2240; and calculations are found to be tedious in such cases when made by the regular methods.

The following rule will be found simple and practical where 2268 pounds to the ton are used:

RULE. *First take one-seventh of the number of pounds, then one-ninth of that seventh; the result will be the cost, in dollars and cents, at* $36 *per ton.* From this the cost at any given price is readily found by aliquot parts, as in the example given at page 195.

EXAM. 1. What is the cost of 1728640 pounds of pig-iron at $12 per ton (2268 lbs.)?

Here we take $\frac{1}{7}$ of the number of pounds, then $\frac{1}{9}$ of that seventh; the result is $27438.73, the cost at $36. Now the cost at $12 is $\frac{1}{3}$ of the cost at $36.	1728640 246948.57 27438.73=$36 =$9146.24=$12

REASON: Instead of using 2268 as divisor, we prefer to make use of the factors, 4, 7, 9 and 9 ($4 \times 7 \times 9 \times 9 = 2268$) (It is immaterial in what order the factors are taken in the multiplication to produce 2268.) If we take 7×9 we have 63, leaving 4×9, or 36. Hence we have $63 \times 36 = 2268$. Now if we divide 2268 by 63, or by 7 and 9 in succession, we get 36.

Cancellation will often enable us to shorten the process, as in the following:

EXAM. 2. What is the cost of 2742643 pounds of iron $14.40 per ton (2268 lbs.)?

~~4~~	274264	3	$1~~4~~.4~~0~~
7	1097057	20	~~3~~.6~~0~~
9	156722	457	.40
~~9~~	$17413	606	

Here we set the factors of 2268 on the left, and the price $14.40 on the right, of the pounds. We eliminate 4, on the left, and dividing $14.40 by 4 also, we get $3.60. Next we eliminate 9, on the left, and dividing $3.60 by 9 also, we get 40c. Multiplying now by .40 we have $1097057.20, and taking $\frac{1}{7}$ of that, and $\frac{1}{9}$ of that seventh, we get $17413.606, the required cost.

NOTE.—To solve this problem according to the rule, take $\frac{1}{7}$; $\frac{1}{9}$ of that seventh, and $\frac{1}{3}$ of that ninth, and you have the cost at \$12. Then \$2 is $\frac{1}{6}$ of \$12 and 40c is $\frac{1}{5}$ of \$2. Or, \$2.40 is $\frac{1}{5}$ of \$12.

And if there be tons and the fraction of a ton proceed as in the two following examples:

EXAM. 3. What is the cost of $40\frac{1872}{2268}$ tons of pig-iron at \$14 per ton?

	$\cancel{4}^2$	1872	$\$\cancel{14}$
Here we find the cost, first of 1872 lbs. to be	$\cancel{7}$	936	
\$11.55, by the method of cancellation as in the	9	104	
preceding example.	9	\$11.55	
Next, we have $40 \times 14 =$ \$560 the cost of 40 tons		=\$560.	
which is added to \$11.55, making total cost.		=\$571.55	

Another example will make this simple method clear.

EXAM. 4. What is the cost of $25\frac{1986}{2268}$ tons of iron at \$13.50 per ton?

Here we first find the cost of 1986 lbs. at \$12,	1986	
as in example 1, by taking $\frac{1}{7}$, $\frac{1}{9}$ and $\frac{1}{3}$; or $\frac{1}{3}$, $\frac{1}{7}$	662	
and $\frac{1}{9}$, getting \$10.508.	94.571	
Then \$1.50 is $\frac{1}{8}$ of \$12, so we take an eighth	\$10.508	=\$12
of \$10.508 to get cost at \$1.50	= 1.313	= \$1.50
Next, we have $\$13.50 \times 25$, or $\$1350 \div 4$	=\$337.50	
Making the total cost	=\$349.321	

And if the price per ton were \$13.75, we would simply add a sixth of \$1.50, 25c being $\frac{1}{6}$ of \$1.50

NOTE.— It is scarcely necessary to remark that multiplying \$13.50 by 25, is the same as multiplying by 100 and dividing by 4; 25 being $\frac{1}{4}$ of 100.

REMARKS.

The careful student need scarcely be told, at this stage of the work, that, in computing the cost of iron, etc., in quantities of *pounds* at so much per ton, much needless labor can be saved by reducing the pounds to tons and the *decimal* of a ton, instead of to tons and *pounds* as is most frequently done.

TO REDUCE POUNDS TO TONS OF 2268 LBS.

Rule. — Take one-fourth of the number of pounds, one-seventh of that fourth, one-ninth of that seventh and one-ninth of that ninth; the last result is the number of tons and the decimal of a ton, and will represent the cost, in dollars and cents, at $1 *per ton always; and if British money, at* £1. *sterling per ton, etc.*

NOTE.—The *reason* for making use of these divisions is explained on page 222.

EXAM. What is the cost of 58668 pounds of iron at $10 per ton (2268 lbs.)?

Taking a quarter of the pounds, we get..............		14667
one-seventh of this quarter	=	2095.2857
one-ninth of the latter	=	232.8095
and one-ninth of the last	=	25.8677

Here we have 25.8677 tons, expressed decimally, instead of 25 tons 1968 lbs. found by dividing by 2268.

At $1 per ton, the cost of 25.8677 tons is................		$25.8677
and at $10 per ton, the cost is ten times that	=	$258.677

NOTE.—Should it be necessary to express the cargo on the invoice, or on the books, in tons and pounds it can be easily done, but the *computations* will be simplified by the method shown above, or better perhaps, in the majority of cases, by the rule given on page 222.

TO REDUCE POUNDS TO GROSS TONS (2240 LBS.)

Rule.—Cut off one figure from the right of the pounds and take one-fourth, one-seventh of that fourth and one-eighth of that seventh; the last result will be the tons and the decimal of a ton, and will represent the cost, in dollars and cents, at $1 per ton, always; and if British money, at £1. sterling per ton, etc.

REASON.—Because 40, 7 and 8, are component factors of 2240 ($40 \times 7 \times 8 = 2240$). Cutting off one figure and dividing by 4 divides by 40; then $\frac{1}{7}$ of that and $\frac{1}{8}$ of the seventh, completes the division by 2240.

EXAM. What is the freight on 169840 lbs. of mdse. at 90c. per gross ton (2240 lbs.)?

Cutting off the right-hand figure		16984	0
and taking the parts as pointed out		4246	
by the rule, we get 75.8214 tons,		606	5714
carried to four decimal places; and at $1 per ton	=	$75	8214
At 10c. the cost is the tenth of that at $1	=	7.5821	
Taking this from the cost at $1, we get the cost at 90c.	=	$68.2393	

Suppose, now, it were required to find the freight on 347 cwt. 3 qrs. 21 lbs. of mdse., in American currency, at 23s. 8d. British, per gross ton (2240 lbs.), the rate being $4.87½.

In 347 cwt. 3 qrs. 21 lbs. there are 38969 lbs.		3896	9 lbs.
Dividing now according to the rule,		974	225
we get 17.3968 tons, carried to four		139	175
decimal places; and at £1. or 20s. per ton the cost	=	£17	3968
3s. 4d. being $\frac{1}{6}$ of £1. or 20s. we take $\frac{1}{6}$	=	2	8994
4d. is a tenth of 3s. 4d. (40d.) $\frac{1}{10}$	=		2899
making the cost at £1. 3s. 8d., or 23s. 8d.	=	£20	5861
At $1 per pound sterling we have $20.5861; at $10	=	$205.861	
Taking half $205.861, the value at $10, we get......		$102.9305	
the value at $5. From this we take $\frac{1}{8}$ of $20.5861	=	2.5732	
or $\frac{1}{40}$ of $102.9305, and we have the required freight	=	$100.3573	

(*See rule, exam. and note, page* 204; *also rules and reason page* 201.)

TO REDUCE GROSS TONS TO NET.

RULE.—*Add to the gross one-tenth of itself, and one-fifth of that tenth.*

EXAM. 1. How many net tons in 39 gross tons?

Adding to the gross one-tenth of itself, or 3.9, and one-fifth of this tenth, or .78, we get the net tons and the decimal of a ton $43.68 = 43\frac{1360}{2000}$, or 43 tons, 1360 lbs.

$$\begin{array}{r} 39 \\ 3.9 \\ .78 \\ \hline 43.68 \end{array}$$

Reason. To reduce gross tons to net, the gross is multiplied by 2240 and the result divided by 2000.

Now, by taking 2000 for multiplier and 2000 for divisor, and eliminating both, the multiplication by 2240 is completed by adding a tenth of 2000 and a fifth of that tenth; 200 being a tenth of 2000, and 40 a fifth of 200.

$$2000 \quad 39 \quad \begin{array}{r} 2000 \\ 200 \\ 40 \end{array}$$

EXAM. 2. How many net tons in 47 tons 15 cwt. gross?

Since there are 20 cwt. in a ton, 15 cwt. $= \frac{15}{20}$ or .75 of a ton; therefore 47 tons, 15 cwt. = 47.75 gross tons and by adding to this one-tenth of itself and one-fifth of that tenth, we obtain 53.48 net tons, or 53 tons 960 lbs. ($.48 \times 2000 = 960$.)

$$\begin{array}{r} 47.75 \\ 4.775 \\ .955 \\ \hline 53.48 \end{array}$$

EXAM. 3. In 56 tons, 1860 lbs. gross, how many net tons?

In this, we find first the net tons in 56 gross as in the foregoing example, by adding a tenth and a fifth of that tenth and we obtain 62.72 net tons.

Next we add to this $\frac{1860}{2000}$ of a ton, which is .93 and we have 63.65 tons net, or 63 tons, 1300 lbs.

$$\begin{array}{r} 56 \\ 5.6 \\ 1.12 \\ \hline 62.72 \\ .93 \\ \hline 63.65 \end{array}$$

EXAM. 4. How many net tons in 480 gross?

Adding one-tenth, and one-fifth of that one-tenth, we obtain 537.6 net tons, or 537 tons, 1200 lbs.

$$\begin{array}{r} 480 \\ 48. \\ 9.6 \\ \hline 537.6 \end{array}$$

NOTE.—To reduce .6 of a ton to pounds, multiply by 2000.

TO REDUCE NET TONS TO GROSS.

RULE. — *From the net deduct one-seventh of itself, and to the result add one-fourth of that seventh.*

EXAM. How many gross tons in 43 tons 1360 lbs. net?

43 tons, 1360 lbs. or $43\frac{1360}{2000}$ tons = ...	43.68
Deducting one-seventh of this, or	6.24
6.24, we get 37.44 to which is	37.44
added one-fourth of 6.24, or 1.56	1.56
getting 39 gross tons.	39.00

Reason. — To reduce net to gross we multiply the net by 2000 and divide by 2240.

By deducting one-seventh from each of the two first terms the relation of these numbers is not changed. Nor is the relation changed by adding to the remainders one-fourth of that seventh; and the process equalizes the first and third terms. In other words, we have now to multiply by 2000 and divide by 2000, which does not affect 39.

2240	43.68	2000
320	6.24	
1920	37.44	
80	1.56	
2000	39.00	

Or, to be more explicit, the numbers 2240 and 43.68 bear to each other the relation of divisor and dividend respectively, and if any change be made in the divisor a similar change must be made in the dividend to preserve the relationship. (*See Gen. Prin. page* 9.)

NOTE. — It need scarcely be remarked that 112 and 100 can be used instead of 2240 and 2000; and also that this method can be made use of in dividing any number by 112 or 2240; first performing the process as above and then dividing by 100 in one case and by 2000 in the other, at the finish.

THE NET COST BEING GIVEN TO FIND THE GROSS.

RULE. — *Add to the net cost one-tenth of itself and one-fifth of that tenth.*

EXAM. If a net ton of coal cost \$4.50, what ought a gross ton cost at the same rate?

Adding one-tenth and one-fifth of that tenth, we get the gross cost, \$5.04.	\$4.50
	.45
	9
	\$5.04

Reason. — Since the cost of a gross ton is more than that of a net, at the same rate, we multiply the net cost by 112 and divide by 100, making use of the short method explained on page 226.

THE GROSS COST BEING GIVEN TO FIND THE NET.

RULE. — *From the gross cost deduct one-seventh of itself, and to the result add one-fourth of that seventh.*

EXAM. If a gross ton of coal cost \$5.04, what ought a net ton cost, at the same rate?

Deducting one-seventh, and adding to the result one-fourth of that seventh we get the net cost, \$4.50.	\$5.04
	.72
	\$4.32
	.18
	\$4.50

Reason. — The reason will be understood from that given on page 227. It is simply a short method of multipiying by 2000 and dividing by 2240.

NET TONS.

In computing the cost of merchandise in pounds, at so much per ton net, operations are simplified by assuming 1c *per pound, or* $20 *per ton, as a standard price always.*

For example, the cost of 23760 pounds of mdse. at 1c per lb. or $20 per ton, net	=	$237.60
And at $2 per ton, the cost is a tenth of $20..........	=	23.760
At 20c the cost is a tenth of that at $2...............	=	2.3760
And at 2c per ton, it is a tenth of 20c................	=	.23760

In other words, by moving the decimal point two places to the left in any number of pounds we have the cost at $20 per ton net, always; moving the point three places to the left gives the cost at $2 per ton; moving the point four places, gives the cost at 20c. and moving the point five places, gives the cost at 2c per ton.

And from this simple basis, the cost at any given price may be readily obtained.

EXAM. What is the cost of 23760 lbs. of soft coal at $2.50 per ton net?

At $2, the cost is		$23.760	Or thus:		
And at 50c it is ¼ of $2	=	5.94	At $20, the cost	=	$237.60
Making the cost at $2.50	=	$29.70	And 2.50 is ⅛ of $20	=	$29.70

NOTE.—If the price were $2.75, we would add for 25c. half that at $50c. or a tenth of that at $2.50. If the price were $1.75 per ton, we would deduct from $23.76 (price at $2) one-eighth of itself, 25c. being ⅛ of $2; and if $2.25. we would add ⅛ of the cost at $2, etc.

Sometimes 2200 pounds are allowed to a ton. In that case, by deducting from 2200 one-eleventh of itself, or 200, it becomes a ton of 2000 pounds; and by making a similar change in the number of pounds to be computed, the foregoing rule for net tons can be applied.

For example, the cost of 1887 lbs. of coal at $2 per ton of 2200 lbs. would be $\frac{1}{11}$ off; then moving the decimal point three places to the left we get $1.715 + and from this the cost at any price.

1887
171.54
$1.71546

NET TONS.

EXAM. What is the cost of 1840 lbs. of hay at $13.75 per ton net?

The cost of 1840 lbs. at $20 per ton..........	=	$18.40
and at $10 it is half that at $20	=	$9.20
At $2.50 it is one fourth of $10	=	2.30
and at $1.25 it is half that at $2.50	=	1.15
Making the cost at $13.75	=	$12.65

Or thus:

At $2 per ton the cost is	=	$1.840
and at $14 it is seven times that at $2	=	$12.88
At 25c. it is an eighth of that at $2	=	.23
which is deducted, giving the cost at $13.75	=	$12.65

NOTE — If the price per ton were 13¾c. = .1375; or $1.37½ = $1.375, the same figuring would answer by simply moving the decimal point one place to the left in each case. And this method of aliquot parts will be found equally simple at any given price per ton by working from the basis given on page 229.

EXAM. What is the cost of 24640 lbs. of beet sugar at $4.26⅔ per ton net?

Here we have the cost at $20 per ton	=	$246.40
and at $4 the cost is one-fifth of that at $20..........	=	$49.28
At 20c. the cost is $2.46, taken from top figures	=	2.46
and at 6⅔c. it is a third of that at 20c.	=	82
giving the cost at 4.26⅔.	=	$52.56

NOTE.—If the price were $4.27⅔ per ton, we would add for 1c. half of 24c. taken at sight from the top figures $246.41; and if $4.28⅔, the addition would be 24c, taken from the top figures, or, rather, 25c, the third figure, 6, being greater than 5 or more than one-half.

GROSS TONS OF RAILS TO THE MILE.

RULE. — *To find the number of gross tons of rails to a mile of railroad: Multiply the number of pounds of rail to the yard by* 11 *and divide by* 7.

EXAM. How many gross tons of rails to a mile of railroad at 60 pounds to the yard?

Here we have $\frac{60 \times 11}{7} = \frac{660}{7} = 94.285$ gross tons.

Reason.— The process in full would be: 8 fur. $\times$ 40 per $\times$ $5\frac{1}{2}$ yds. multiplied by 60 $\times$ 2 (two rails to track), and the result divided by 2240 (lbs. to the gross ton), or by 40 $\times$ 7 $\times$ 8, the component factors of 2240. Here then we have the fraction $\frac{8 \times 40 \times 5\frac{1}{2} \times 60 \times 2}{40 \times 7 \times 8}$; and by cancellation $\frac{\cancel{8} \times \cancel{40} \times 5\frac{1}{2} \times 60 \times 2}{\cancel{40} \times 7 \times \cancel{8}} = \frac{11}{7} \times 60.$

NET TONS OF RAILS TO THE MILE.

RULE. — *To find the number of net tons of rails to a mile of railroad: Multiply the number* 1.76 *by the number of pounds of rail to the yard.*

EXAM. How many net tons of rails to a mile of road at 60 pounds to the yard?

Here we have $1.76 \times 60 = 105.60$ net tons.

Reason. — The process in full would be 1760 yds. $\times$ 60 $\times$ 2, divided by 2000; or $\frac{1760 \times 60 \times 2}{2000} = \frac{1760 \times 60 \times 1}{1000} = 1.76 \times 60.$

There are 1760 lineal yards to a statute mile, or, 8 fur. $\times$ 40 per. $\times$ $5\frac{1}{2}$ yds. = 1760 yds.

TRADE DISCOUNTS; SHORT METHODS.

Manufacturers and wholesale dealers usually allow to the *trade* or retail dealers, a reduction from the fixed or *list prices* of some kinds of merchandise. This reduction is called a discount, or a *trade discount*. In some lines of business several discounts are allowed. Discounts are taken off in succession. Thus, 25%, 15% and 10% off; or, as it is generally expressed in business, 25, 15 and 10 off, means first a discount of 25%, then 15% of what is left, and finally, 10% of the remainder.

NOTE. — The profit on goods is less when 25, 15 and 10 are allowed, than if 50 per cent were allowed.

It is immaterial in what order the discounts are taken as it will not affect the result.

EXAM. 1. If goods be listed at \$65 with 40% off, what is the net cost?

Usual Method.			Short Method.		
$\$65 \times .40$	=	\$26	$\$65 \times .60$	=	\$39 net.
then $65 - 26$	=	\$39			

Instead of multiplying the list-price, \$65, by .40, and deducting the result, it is much shorter to multiply by .60, the difference between 1.00 and .40, which gives the net at once; or, as it is said in the trade 40 off is 60 on. And this short rule will hold good for any series of discounts.

NOTE. — Since any per cent. is some number of hundredths, it is properly expressed by a *decimal fraction*; thus 5 per cent. $= 5\% = .05$.

It is scarcely necessary to remark that the list-price represents 100 per cent. $= 100\% = \frac{100}{100} = 1.00 = 1$. Now, if from this we deduct $40\% = .40$, the difference will be $60\% = .60$; so we multiply the list-price by .60 straight to get 40% off.

TRADE DISCOUNTS; SHORT METHODS.

EXAM. If goods be listed at \$3 with 30, 20 and 10 off, what is the net cost?

\$3 .7 ——— 2 1 .8 ——— 1.68 .9 ——— \$1.512	Or thus: $.7 \times .8 \times .9 = .504$ 3 ——— \$1.512

Here we deduct, at sight, 30, 20 and 10, each, from 100% getting 70%, 80% and 90%; or, when decimally expressed, .70, .80 and .90; or, .7, .8 and .9 (the ciphers not affecting the significant figures in the decimal expressions). Multiplying by .7, .8 and .9 in succession, we get \$1.512 the net. Or, multiplying .7, .8 and .9 together, we get $.504 = 50\frac{4}{10}\%$ which is the net discount equal to 30, 20 and 10; and multiplying this net by the list price we get the net cost.

NOTE 1. To find the net rate of discount equal to several rates: *Deduct each rate from 100, multiply the differences together and the product is the net rate.* Thus, 60%, 25% and 10% off, is equal to 27% net, or to 73% off $(.40 \times .75 \times .90 = .27)$; or, $(.4 \times .7\frac{1}{2} \times .9 = .27)$.

NOTE 2. It should be carefully borne in mind that the cipher, or zero, having no value, is used in combinations of figures to fill places where no value is to be expressed, and thus to make the other figures occupy those places in which they will express the intended values. Hence, a cipher will not affect the value of a number *unless it be placed between some significant figure and the decimal point.* Thus, .7, .70, .700, .7000, etc., are all equal in value. But .07, .007, .0007, are entirely different, the *local value* of 7 being changed by the cipher, or ciphers coming between that figure and the decimal point. Briefly, then, *the use of the cipher is to keep the significant figures in proper position with reference to the decimal point.*

The use of the decimal point is to mark the place of units; and whether expressed or understood, its position is always to the *right* of units. It is a matter of the utmost importance to have a correct knowledge of making a proper use of the decimal point in our calculations.

TRADE DISCOUNTS; SHORT METHODS.

Given the net cost and the discount to find the list-price.

RULE. — *Divide the net cost by the net discount.*

EXAM. If the net cost of goods be \$1.512, and the discounts are 30, 20 and 10 off, what is the list-price?

Multiplying .7, .8 and .9 together we get .504, the net discount. Then \$1.512 ÷ .504; or, moving the decimal point three places to the right in these numbers, in other words, multiplying each by 1000, to throw off the decimals, we have \$1512 ÷ 504 = \$3, the list-price.

Reason. — Since the net cost is found by multiplying the list-price by the net discount, we simply reverse the rule here, viz.; divide the net cost by the net discount to find the list-price.

OTHER SHORT METHODS.

In marking goods, merchants generally take a rate per cent. that is an aliquot part of 100, as 50, $33\frac{1}{3}$, 25, $12\frac{1}{2}$, $8\frac{1}{3}$, etc.; and instead of multiplying by the net discounts, in such cases, to find the net cost, it will be found preferable to make use of the method of aliquot parts, as in the following

EXAM. If goods be listed at \$240 with $33\frac{1}{3}$, 25 and 5 off; what is the net cost?

Here, instead of deducting $33\frac{1}{3}$, 25 and 5, each, from 100, and multiplying by the net discounts, $.66\frac{2}{3}$, .75 and .95, we take $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{20}$ off in succession ($33\frac{1}{3} = \frac{1}{3}$ of 100; $25 = \frac{1}{4}$ and $5 = \frac{1}{20}$), thus getting the net cost, \$114.

$$\begin{array}{r} \$240 \\ 80 \\ \hline 160 \\ 40 \\ \hline 120 \\ 6 \\ \hline \$114 \end{array}$$

NOTE. — To find the aliquot parts of 100 divide it by 2, 3, 4, 5, etc. Thus, $100 \div 6 = 16\frac{2}{3}$, and $16\frac{2}{3}$ is 1-6 of 100, etc.

TRADE DISCOUNTS; SHORT METHODS.

Odd rates of discount will be found equally simple in many cases where the inexperienced calculator has to make use of long methods.

EXAM. If goods be listed at \$148 with $47\frac{1}{2}$ off; what is the net?

Here instead of multiplying by $.52\frac{1}{2}$, we say 50 is a half, and $2\frac{1}{2}$ is $\frac{1}{20}$ of 50, or $\frac{1}{40}$ of 100; and adding, we have multiplied by $.52\frac{1}{2}$.

$$\begin{array}{r} \$148 \\ \hline 74 \\ 3.70 \\ \hline \$77.70 \end{array}$$

NOTE.—And if the rate were $52\frac{1}{2}$ off, instead of $47\frac{1}{2}$, the same figuring would answer, only instead of adding 3.70 we deduct ($100 - 47\frac{1}{2} = 52\frac{1}{2}$); ($50 + 2\frac{1}{2} = 52\frac{1}{2}$); ($50 - 2\frac{1}{2} = 47\frac{1}{2}$). And if the rate were $37\frac{1}{2}$, we would say $25 = \frac{1}{4}$ and $12\frac{1}{2} = \frac{1}{2}$ of 25 ($25 + 12\frac{1}{2} = 37\frac{1}{2}$); $27\frac{1}{2}$ off would be $25 = \frac{1}{4}$ of 100 and $2\frac{1}{2}$ a tenth of 25; and so of other rates.

EXAM. If goods be listed at \$420 with 65% off; what is the net?

Here we say 65 off equals 35 on and multiplying \$420 by .35 we get \$147 net.

$$\begin{array}{r} \$42.0 \\ 105. \\ \hline \$147 \end{array}$$

For short method we take 10 and 25; 10% = \$42, at sight, and 25% = $\frac{1}{4}$ of 420, or 105 which is added in proper position, one place to the left.

To make the process clearer it must be borne in mind that \$420 represents 100% .... = \$420
then 25% = $\frac{1}{4}$ of 100 .... = 105
and 10% = $\frac{1}{10}$ of 100 .... = 42
Making 35% (25 + 10); or (10 + 25 = 35) . ... = \$147

TRADE DISCOUNTS; SHORT METHODS.

EXAM. What is the net cost of goods which are listed at $6.40 with a discount of $62\frac{1}{2}\%$ off?

Here we have $100 - 62\frac{1}{2} = 37\frac{1}{2}$, the net		$6.40
discount. We now say $25 = \frac{1}{4}$	=	$1.60
and $12\frac{1}{2} = \frac{1}{2}$ of 25, or $\frac{1}{8}$ of 100	=	80
Adding both results we have multiplied by $37\frac{1}{2}$		$2.40

Or, simply take $\frac{1}{8}$ of $6.40 = 80c. and three times 80c. gives $2.40, the net cost, $37\frac{1}{2}$ being $\frac{3}{8}$ of 100.

NOTE.—If the rate were $37\frac{1}{2}$ off, then the net would be $62\frac{1}{2}$, and to multiply by $62\frac{1}{2}$ we would say $50 = \frac{1}{2}$, and $12\frac{1}{2} = \frac{1}{4}$ of 50. Or, take $\frac{1}{8}$ and multiply by 5, $62\frac{1}{2}$ being $\frac{5}{8}$ of 100.

EXAM. If goods be listed at $54 with $67\frac{1}{2}$ off; what is the net cost?

Here $100 - 67\frac{1}{2} = 32\frac{1}{2}$, the net discount	$54 × .3¼
which is equal to $30 + 2\frac{1}{2}$. To multiply by $.32\frac{1}{2}$	$16.2
we multiply first by .30, and for $2\frac{1}{2}$ we add	1.35
$\frac{1}{40}$ of $54, or $\frac{1}{4}$ of $5.4 ($5.40), $2\frac{1}{2}$ being $\frac{1}{40}$	$17.55

of 100, or $\frac{1}{4}$ of 10. Or, by looking on $.32\frac{1}{2}$ as $.3\frac{1}{4}$, simply multiply by $.3\frac{1}{4}$ bearing in mind that the quarter in such cases is in reality $\frac{1}{40}$ of the list price, or $\frac{1}{4}$ of its tenth.

NOTE.—If the rate were $32\frac{1}{2}$; then the net would be $67\frac{1}{2}$, and to multiply by $.67\frac{1}{2}$ we would multiply first by .60, and to the result add its $\frac{1}{8}$, $7\frac{1}{2}$ being $\frac{1}{8}$ of 60.

Or, take 50, 5 and $12\frac{1}{2}$ ($50 + 5 + 12\frac{1}{2} = 67\frac{1}{2}$.)

And other odd rates of discount will be found equally simple. If the rate were $57\frac{1}{2}$ off, for instance, the net would be $.42\frac{1}{2}$. To multiply by $.42\frac{1}{2}$ look upon it as $.4\frac{1}{4}$, bearing in mind that the quarter is $\frac{1}{40}$, as in the foregoing example. If the net were $57\frac{1}{2}$ we would resolve it into 50, 5 and $2\frac{1}{2}$; $50 = \frac{1}{2}$; $5 = \frac{1}{10}$ of 50, and $2\frac{1}{2} = \frac{1}{2}$ of 5.

TRADE DISCOUNTS; SHORT METHODS.

EXAM. If goods be listed at \$80 with 40, 10, 10, 5, 5, $7\frac{1}{2}$ and 3 off; what is the net cost?

Since the rates of discount may be taken off in any order without affecting the result, we begin here with $7\frac{1}{2}$ and find the net rate as follows:

Deducting $7\frac{1}{2}$ from 100 we get $92\frac{1}{2}$, and express it decimally, .925 To get 3% off we simply set 3 times .925 two places to the right and subtract. Then $\frac{1}{20}$ of the result is subtracted, and $\frac{1}{20}$ of what is left respectively, for the two 5's, giving .80978. Next, 10% is taken off by

			100
			$7\frac{1}{2}$
$7\frac{1}{2}$ off		=	.925
			2775
3	"	=	.89725
			4486
5	"	=	.85239
			4261
5	"	=	.80978
10	"	=	.72881
10	"	=	.65593 × .6
40	"	=	.393558 × 80
			\$31.48464

subtracting each left hand figure from the one immediately to the right, beginning with the tens, or second figure in .80978, thus: 7 from 8 leaves 1; 9 from 17 leaves 8; carry 1 to 0; 1 from 9 leaves 8; 8 from 10 leaves 2; 1 to carry taken from 8 leaves 7, the result is .72881. And from this number the second 10% is got in like manner, giving 65593. Then 40% is taken off by multiplying .65593 by .6, the net equivalent to .40; the product, .393558 is the net rate. This is now multiplied by 80, the list price, and we get \$31.48 +, the net cost.

NOTE.—The net rate, .393558 is nearly equal to $.39\frac{35}{100}$, or $39\frac{7}{20}\%$, and is the product of $.92\frac{1}{2} \times .97 \times .95 \times .95 \times .90 \times .90 \times .60$, the multiplication being performed much more simply by the process given above.

By taking $39\frac{7}{20}\%$ from 100 we get $60\frac{13}{20}\%$ which is the rate of *discount off* equivalent to the seven rates, 40, two 10's, two 5's, $7\frac{1}{2}$ and 3 off. (*See page* 233, *also note* 1, *same page.*)

INTEREST REVIEWED.

Since the interest of \$100 for 1 year at 1% = \$1.
the interest of \$100 for 100 years at 1% = \$100.
in other words, money doubles, or the interest
will equal the principal in 100 years, at 1%, simple interest.

If, then, 100 years be divided by the rate per cent. it will give the time, in years, when money doubles at that rate, simple interest.

Thus, any sum of money at 2%, will double itself in 50 years; at 4%, in 25 years; 5%, in 20 years; 6%, $16\frac{2}{3}$ years, &c.

Hence, taking 100 years and 1% for the basis, and reducing the 100 years to months and days, we have the time in which

money doubles at			1 %	=	36000	days	=	1200	mo.	=	100	years.
"	"	"	2 %	=	18000	"	=	600	"	=	50	"
"	"	"	$2\frac{1}{4}$%	=	16000	"	=	$533\frac{1}{3}$	"	=	$44\frac{4}{9}$	"
"	"	"	3 %	=	12000	"	=	400	"	=	$33\frac{1}{3}$	"
"	"	"	$3\frac{3}{4}$%	=	9600	"	=	320	"	=	$26\frac{2}{3}$	"
"	"	"	4 %	=	9000	"	=	300	"	=	25	"
"	"	"	$4\frac{1}{2}$%	=	8000	"	=	$266\frac{2}{3}$	"	=	$22\frac{2}{9}$	"
"	"	"	5 %	=	7200	"	=	240	"	=	20	"
"	"	"	6 %	=	6000	"	=	200	"	=	$16\frac{2}{3}$	"
"	"	"	$7\frac{1}{2}$%	=	4800	"	=	160	"	=	$13\frac{1}{3}$	"
"	"	"	8 %	=	4500	"	=	150	"	=	$12\frac{1}{2}$	"
"	"	"	9 %	=	4000	"	=	$133\frac{1}{3}$	"	=	$11\frac{1}{9}$	"
"	"	"	10 %	=	3600	"	=	120	"	=	10	"
"	"	"	12 %	=	3000	"	=	100	"	=	$8\frac{1}{3}$	"
			&c.		&c.			&c.			&c.	

NOTE.--It will be seen, on examining the foregoing, that the time is proportional to the rate, and vice versa; thus, the interest of any sum of money for 18000 days at 2%, is equal to the interest of that sum for 9000 days at 4%; the interest at 9% for 4000 days, is equal to that at $4\frac{1}{2}$% for 8000 days, or $2\frac{1}{4}$% for 16000 days; and the interest of any sum for 4800 days at $7\frac{1}{2}$%, is equal to the interest for 9600 days at $3\frac{3}{4}$%, &c

A careful analysis of the matter given on the foregoing page, will now enable us to compute interest at any rate, and for any time, with ease and rapidity. To do so, we must keep in view the basis, or the time in which money doubles at 1%, simple interest, viz., 36000 days, 1200 mo. or 100 years.

Take any particular rate, say $2\frac{1}{4}$%, for example, and suppose it were required to find the interest of $5764.50 for 16 days, at that rate.

Here we simply point off three figures to the right, counting from the decimal point, and we get $5.7645, the required interest, true to four places of decimals. $5 7645

Reason.—Since money doubles in 36000 da. at 1%, simple interest, it will double at 2¼% in 16000 da.; 2¼ being contained 16000 times in 36000. Hence,

The interest of $5764.50 for	16000 da.	at 2¼%	=	$5764.50
one tenth of this is the interest for	1600 "	"	=	$576.45
one tenth of the latter is the int. for	160 "	"	=	$57 645
and a tenth of this last is the int. for	16 "	"	=	$5.7645

We have here, now, a basis to find the interest for any number of days, at $2\frac{1}{4}$%. If 40 days, take $\frac{1}{4}$ of 160 days' interest, or of $57.645; the interest is $14.41; if 80 days, the interest will be half of $57.645, or $28.82; if 32 days, it will be two times 16 days' int. or $11.529; if 10 days' interest be required, take $\frac{1}{2}$ of 160 days' interest, then $\frac{1}{8}$ of that half; or, take $\frac{1}{4}$ of 160, then $\frac{1}{4}$ of that fourth; the result will be 10 days' interest.

If the time be 3 mo. 10 da. or 100 days, we have 6 times 16 da. equal 96, plus 4 da., or $\frac{1}{4}$ of 16; and if 4 mo. or 120 da., we have 160 da. less a quarter of that, or 40 days' interest, the difference will be 4 mo., or 120 days' interest, &c.

And if one day's interest be required at $2\frac{1}{4}$%: point off three figures from the right, counting from the decimal point, the result is 16 days' interest always. Then take $\frac{1}{2}$ and $\frac{1}{8}$ of that $\frac{1}{2}$ ($2\times8=16$.) or, take $\frac{1}{4}$ and $\frac{1}{4}$ of that $\frac{1}{4}$ ($4\times4=16$.)

Again, suppose it were required to find the interest on a loan, say, of \$300,000 for 1 day, at $3\frac{3}{4}$%.

By referring to page 238, we see that $3\frac{3}{4}$ is contained 9600 times in 36000 days.

Now, since the interest will equal the principal in 9600 days, at $3\frac{3}{4}$%, simple interest, always:

We have, here, at sight, the interest for	9600 da.	=	\$300000
one tenth of this, or the interest for	960 "	=	\$30000
one tenth of the latter, or the interest for	96 "	=	\$3000
and $\frac{1}{96}$ of this last is the interest for	1 "	=	\$31.25

Hence,

RULE.—*To find the interest of any sum for* 1 *day, at* $3\frac{3}{4}$% : *Move the decimal point two places to the left, in other words, take* 1% *of the principal, and the result is* 96 *days' interest, always. Divide this by* 96 *for* 1 *day's interest.*

EXAM. What is the interest of \$376860.48 for 1 day at $3\frac{3}{4}$% ?

Taking 1% of the principal we have	\$3768.6048	=	96 da. int.
one twelfth of \$3768.6048 gives	\$314.0504	=	8 " "
and one eighth of \$314.0504 gives	\$39.2563	=	1 " "

NOTES.—1. It will be observed that, in dividing by 96, to get 1 day's interest, we have made use of 12 and 8, the component factors of that number ($12 \times 8 = 96$.) And in all such cases where the divisor can be readily factored this method of division is to be preferred. Thus, to get 1 day's interest at $7\frac{1}{2}$%, we would divide 1% of the principal by 48 (the significant figures in 4800 days, or the time in which money doubles at $7\frac{1}{2}$%, simple interest), making use of the factors, 6 and 8; and if 5%, we would divide 1% of the principal by 72 (the significant figures in 7200 days, the time in which money doubles at 5%, simple interest), using 8 and 9, or 6 and 12, the factors of 72, &c.

2. If we examine carefully the foregoing example, it will be readily seen how easily the interest for any number of days may be obtained at $3\frac{3}{4}$%. If the time were 48 days, for instance, instead of multiplying 1 day's interest by 48, we would simply take $\frac{1}{2}$ of 96 days' interest, or 6 times 8 days' interest. 32 days' interest is $\frac{1}{3}$ of 96 days, or 4 times 8 days' interest; 33 days would be 32 plus 1 day, and 31 days would be 32 minus 1 day. By adding 1 day and 8 days' interest we have 9 days, and by deducting 1 from 8 we have 7 days, &c.

It will now be seen that, when the rate is an exact divisor of 36000 days, or 1200 mo., the computation of interest can be made simple and interesting. And this being properly understood, computations will be found equally simple when the rate is not an exact divisor. Take for instance, the following

EXAM. What is the interest of $75684 for 1 day, at $2\frac{3}{4}\%$?

Since $2\frac{3}{4}$ is not an exact divisor of 36000, we take 3% which is contained 12000 times in 36000; and since the interest of any sum for 12000 days at 3%, is equal to the principal, the interest for 12 days is $\frac{1}{1000}$ of the principal; and this is found by simply moving the decimal point three places to the left in the given principal always.

Here, then, we have 12 days' interest...........	=	$75.684
and $\frac{1}{12}$ of this is the interest for 1 day at 3%........	=	$6.307
Now, $\frac{1}{4}\%$ is $\frac{1}{12}$ of 3%, and we deduct $\frac{1}{12}$............	=	.525
making the interest for 1 day at $2\frac{3}{4}\%$.....	=	$5.782

NOTE.—If the rate were $3\frac{1}{4}\%$, we would, of course, add $\frac{1}{12}$ instead of subtracting; and if $3\frac{1}{2}\%$ then $\frac{1}{6}$ of 3% would be added, $\frac{1}{2}$ being $\frac{1}{6}$ of 3, &c. Hence,

RULE. *To find the interest of any sum for* 1 *day, at any rate per cent.: Divide* 36000 *days by the rate, if it be an exact divisor; divide the given principal by the result, and the quotient is* 1 *day's interest. If the rate be not an exact divisor, take the nearest rate which is an exact divisor, find the interest at that rate, and add, or subtract, for the difference.*

If the rate be 4%, for example, 4 is contained 9000 times in 36000 and we divide the principal by 9000 to get 1 day's interest. If $4\frac{1}{2}\%$, the divisor is 8000; if 6%, the divisor is 6000; and if 5%, it is 7200, &c.

In dividing, point off as many figures from the right, in the given principal, as there are ciphers in the divisor, *counting from the decimal point always.* Thus, to divide by 9000, point off three figures and divide by 9; for 6000, point off three and divide by 6. In dividing by 4500 for 8%, point off two figures and divide by 45, using the factors 5 and 9; first taking $\frac{1}{5}$ of the principal, then $\frac{1}{9}$ of that fifth, making use of short division in all such cases as pointed out in the example given on page 240.

The 6% method is used in most of the Banking Institutions of the country (when interest tables are not used), and is taught in the majority of Commercial Colleges, as the shortest method of computing interest, a very excellent method, indeed (given on p. 180 of this work.)

In adhering to this method, however, a good deal of unnecessary labor has to be gone through in many cases where a knowledge of the foregoing methods will frequently give the interest, at sight, without figuring, at all.

Take, for instance, the following problem at 4½%, and compute the interest on the basis of 6%.

EXAM. What is the interest of \$8000 for 149 days at 4½%?

NOTE.—As a general rule 60 days' interest is taken as the basis of calculation, and the solution, by the 6% method, would be something like the following:

Pointing off two places, we have....	\$80.00	=	60 da. int.
multiplying this by 2, we have........	\$160.00	=	120 " "
one third of 60 days' interest, or......	26.66	=	20 " "
one tenth of 60 days' interest, or......	8.00	=	6 " "
and one half of 6 days' interest, or....	4.00	=	3 " "
giving the interest for 149 da. at 6% =	\$198.66	=	149 " "
From this we deduct ¼ to get.........	49.66		
4½%; 1½% being ¼ of 6%...............	\$149.00	=	int. at 4½%.

NOTE.—Too many figures even for the 6% method. Better point off one place, and we have \$800.0 = 600 days' int. at 6%; 150 da. = ¼ of 600; less 1 day, or ⅙ of \$8; the difference is 149 days' int. at 6%. Then ¼ off is 4½%.

But either method is too long in this case if it be borne in mind that the interest of \$8000 for 149 days, is the same as the interest of \$149 for 8000 days, and that 8000 days is the basis; in other words,

that the interest will equal the principal in 8000 days, at $4\frac{1}{2}\%$, simple interest, always.

Hence,

The problem can be reversed so as to read: What is the interest of $149 for 8000 days, at $4\frac{1}{2}\%$?

The answer is at sight, namely, $149.

NOTE.—This method of reversing the problem can always be used *when it is more convenient to take the dollars for the days.*

If, for instance, it were required to find the interest of $1000 for 149 days, at $4\frac{1}{2}\%$, we would reason thus:

The interest of $1000 for 149 days being	=	$149 for 1000 da.,
and since the interest of $149 for 8000 da.	=	$149
$\frac{1}{8}$ of this will be the interest for 1000 "	=	$18.625

And, in like manner, the interest of $2000, $3000, $4000, $5000, $6000, $7000, $9000, $10000, $12000; or any multiple, as $20000, $25000, $48000, &c., or any part, as $4, $40, $400, &c., can be readily obtained.

To make this important method more clear let us take another

EXAM. What is the interest of $4500 for 5 mo. 17 da. at 8%?

In 5 mo. 17 da. there are 167 days.

Now, since money doubles in 4500 days, at 8%, simple interest, and that the interest of $4500 for 167 da. = that of $167 for 4500 da. the required interest is at sight, viz., $167.

NOTE.—If the principal were $2250, the interest would be ½ of $167 = $83.50; if $9000, the interest would be twice $167, or $334; if $1500, the interest would be ⅓ of $167, or $55.67; and if $15000, the interest would be 10 times that, &c.

And if the rate were 7%, we would, in this case, deduct $\frac{1}{8}$ of the int. at 8%.

Since 7 is not an exact divisor of 36000; interest can be computed at 6% and $\frac{1}{6}$ added, or at 8% and $\frac{1}{8}$ deducted, whichever is most convenient.

Should the time be given in months; years and months; or years, months and days, it will be found preferable, in many cases, to take the time in which money doubles, in *months*, as the basis of calculation, instead of days. Thus:

EXAM. What is the interest on a bond of \$5000 for 7 mo. 9 da., at 5%?

NOTE.—Instead of taking 7200 days, the time in which money doubles at 5%, for the basis of calculation, we prefer to take 240 mo, its equivalent; and since the interest of \$5000 for 240 mo. at 5%, is \$5000, the interest for 24 mo. is $\frac{1}{10}$ of that, or \$500.

Here, then, by pointing off one place	\$500.0	=	24 mo.
in \$5000, we have 24 mo. interest, or \$500	\$125.	=	6 "
6 mo. = $\frac{1}{4}$ of 24 mo., $\frac{1}{4}$ of \$500 = \$125	\$20.83	=	1 "
1 mo. = $\frac{1}{6}$ of 6 mo.; $\frac{1}{6}$ of \$125 = \$20.83;	\$6.25	=	9 da.
and for 9 days, we take $\frac{1}{8}$ of \$50, or 72 da.	\$152.08		

interest, which is always at sight by pointing off two places in the principal, \$5000. Or, to get 9 da. interest we could take 6 da. = $\frac{1}{5}$ of 1 mo. and 3 da. = $\frac{1}{2}$ of 6. Or, better thus, perhaps:

Reducing the time to days, we have 7 mo. 9 da. = 219 da. and since the interest of \$5000 for 219 days, is the same as the interest of \$219 for 5000 days, the problem can be reversed so as to read: The interest of \$219 for 5000 da., at 5%, and the solution be obtained as follows:

The interest of \$219 for 7200 da., at 5%, being....	=	\$219
10 times this will be for 72000 "	=	\$2190

Now, since the interest for	72000 da.	=	\$2190
$\frac{1}{12}$ of this interest will be that for	6000 "	=	\$182.50
$\frac{1}{6}$ of this will be the interest for	1000 "	=	\$30.42
and the difference will be for	5000 "	=	\$152.08

RULE. *When the rate of interest changes frequently: Find the rate for* 1 *day, and compute the interest on the given principal at that rate.*

EXAM. What is the interest on a loan of \$35000 made on Jan. 1, and paid off on the 15th, the rate of interest changing as follows:

Jan. 1	$2\frac{1}{2}\%$
3	2%
6	3%
7	$3\frac{1}{2}\%$
9	3%
10	$3\frac{1}{2}\%$
12	2%
15 paid off.	

Here \$35000 bears interest from the 1st. to the 3rd, or two days at $2\frac{1}{2}\%$; next, 3 days at 2%, &c. Now, $2\frac{1}{2}\%$ for 2 days = 5% for 1 day, &c.

Hence the process:

$$\begin{array}{l}
2\frac{1}{2} \times 2 = 5 \\
3 \times 2 = 6 \\
3 \times 1 = 3 \\
3\frac{1}{2} \times 2 = 7 \\
3 \times 1 = 3 \\
3\frac{1}{2} \times 2 = 7 \\
3 \times 2 = 6 \\
\hline
\qquad 37\% \text{ for 1 day.}
\end{array}$$

We now compute the interest on \$35000 for 1 day at 37%, or, what amounts to the same thing, the interest for 37 days at 1%.

And since the interest of \$35000	for 36000 days	at 1%	= \$35000
" " " "	" 36 days	" "	= \$35
" " " "	" 1 day	" "	= .97
giving the interest required	" 37 days	" "	= \$35.97

NOTE.—In getting 1 day's int. take $\frac{1}{6}$ of \$35; then $\frac{1}{6}$ of the result, making use of the factors, 6 and 6 ($6 \times 6 = 36$.)

ANNUAL INTEREST.

RULE. *When interest is payable annually:*

I. *Compute simple interest of the principal from its given date to the date of settlement.*

II. *Add to this the interest of each year's interest from the time of its accruing to the date of settlement.*

EXAM. What is the amount due, in 3 years, 6 months, on a note of $8000, interest payable annually, at 5%?

$\$8000 \times .05 =$	$400	=	Interest for 1 year.
$\$400 \times 3\frac{1}{2} =$	$1400	=	" " $3\frac{1}{2}$ years.
$\$400 \times .05 \times 2\frac{1}{2} =$	$50	=	" on the 1st year's interest
$\$400 \times .05 \times 1\frac{1}{2} =$	30	=	" " 2nd " "
$\$400 \times .05 \times \frac{1}{2} =$	10	=	" " 3rd " "
	8000	=	Principal.
	$9490		

In this the interest of	$8000 for	$3\frac{1}{2}$ years,	at 5%	=	$1400
Next, the interest of	$400 "	$2\frac{1}{2}$ "	"	=	50
then " " "	" "	$1\frac{1}{2}$ "	"	=	30
finally " " "	" "	$\frac{1}{2}$ year	"	=	10
Making the entire interest.................				=	$1490

NOTE.—It will be seen that the interest of $400 for $2\frac{1}{2}$ years, $1\frac{1}{2}$ yrs. and $\frac{1}{2}$ yr., is equal to the interest of $400 for $4\frac{1}{2}$ yrs., their sum. Thus: $\$400 \times .05 \times 4\frac{1}{2} = \$90$.

PARTIAL PAYMENTS.

A Partial Payment is payment in part of a note, mortgage, bond, or other obligation.

An Indorsement is an acknowledgment of payment, written on the back of the note, mortgage, &c., stating the time and amount of the payment made on the obligation.

NOTE.—The United States Rule, given on page 129 of this work, has been adopted by nearly all the states of the Union, to secure uniformity in the method of computing interest where partial payments have been made on bonds, mortgages and other obligations. It may, however, be proper, here, to give the reader a knowledge of the method generally used in business for settling notes and interest accounts, and is called

THE MERCHANTS' RULE.—I. *Compute the interest of the principal from its date to the time of settlement, and add it to the principal.* II. *Compute the interest of each payment from its respective date to the time of settlement and add this interest to the payments.*

III. *From the amount of the principal take the amount of the payments, and the remainder will be the balance due.*

EXAMPLE.

$5000 Albany, N. Y., Aug. 1, 1902.

Sixty days after date I promise to pay to A. B., or order, five thousand dollars, with interest, value received.

Y. Z.

Indorsed as follows: Nov. 1, 1902, $500; Feb. 1, 1903, $500; April 1, 1903, $1000; July 1, 1903, $2000. How much was due Aug. 1, 1903?

PROCESS:

Principal	=	$5000.
1 yr. int. at 6%	=	300
		$5300
Deduct		4067.50
Balance due	=	$1232.50

Payment	=	$500
9 mo. int. at 6%	=	22.50
Payment	=	500.
6 mo. int. at 6%	=	15.
Payment	=	1000.
4 mo. int. at 6%	=	20.
Payment	=	2000.
1 mo. int. at 6%	=	10
		$4067.50

In long operations in division, or where the same divisor is to be frequently used, *it will be found advantageous to form a table of the several products of the divisor and the nine digits.* Thus, to divide 1709749265 by 365, say to four places of decimals, we look in the table and find that 1460, the product by 4, is the nearest below 1709; we place 4 in the quotient, and, without setting down 1460, subtract it from 1709, setting down the remainder 249, to which 7 is brought down making 2497. We then see in the table that 2190, the product by 6, is the nearest below 2497; 6 is put in the quotient, and 2190, the product by 6, is the nearest below 2497; 6 is put in the quotient, and 2190 is subtracted from 2497 and the remainder is 307, to which 4 is brought down, etc.

1...365)	1709749265(4684244.5616
2...730	2497
3..1095	3074
4..1460	1549
5..1825	892
6..2190	1626
7..2555	1665
8..2920	2050
9..3285	2250
	600
	2350

To divide by 365, however, we prefer to proceed as follows:

Annexing a cipher, and doubling the result, we have 34194985300. To this we add its third, one tenth of that third and one tenth of that tenth: the sum is 46847129860, which we divide by 10001, according to rule II, page 15.

To divide by 10001, we simply cut off four figures to the right for 10000; then *deduct* once 4684712, the figures on the left of the line, and to the result *add* 468, found on the left of the line, also; the quotient is 4684244.5616, true to four places of decimals. (*See rule* II, *page* 15.)

17097492650	
34194985300	
11398328433	
1139832843	
113983284	
4684712	9860÷10001
468	4712
4684244	5148
	468
	.5616

Reason.— By annexing a cipher to 365 and doubling the result; then adding to this its third, one tenth of that third and one tenth of that tenth, we obtain 10001 for a simple divisor, thus establishing a simple method for dividing by 365, always. (*See examples pp.* 51, 52 *and* 53.)

3650
7300
$2433\frac{1}{3}$
$243\frac{1}{3}$
$24\frac{1}{3}$
10001

By simple divisors we mean 10, 100, 1000, 10000, etc., or any number which is a little more, or a little less, as 103, 1008, 1040, etc., 89, 989, 91, 94, 995, 9996, etc., also, the multiples, submultiples and aliquot parts of these numbers.

Divisors may be simplified by any process that will make them 10, 100, 1000, 10000, etc. Thus, to divide by $7\frac{1}{2}$, 7.5, 750, 7500, etc.: add to each its third, and the numbers become 10, 10, 100, 1000, 10000, etc.

EXAM. 1. In 2592 pounds of oil, how many gallons, $7\frac{1}{2}$ pounds to the gallon?

Here, instead of dividing 2592 by $7\frac{1}{2}$, we add to each its third, and we have $3456 \div 10$, which gives 345.6, the required number of gallons.

2592
864
345.6

EXAM. 2. If a gross of articles cost \$237.60, what is the cost of a single article?

Instead of dividing by 144 (articles in a gross) we multiply it by 7, getting 1008 for a simple divisor, then, multiplying \$237.60, also, by 7, we have $1663.20 \div 1008$; and the answer is \$1.65. (*See the rule and exam. page* 57.)

\$1	663.20÷1008
	8
.65	520
	520

EXAM. 3. Divide 35886432 by 9524.

By setting half of each number, in this example, one place to the right, and adding, we have $376807536 \div 100002$; the quotient is 3768.

And if 4762 were the divisor; 2 times that number set one place to the left, and added, would make 10002.

35886432		9524
17943216		4762
3768	07536÷	10002
	7536	

EXAM. 4. Divide 3229184 by 476.

Setting 2 times each term one place to the left, and adding, we get $67812864 \div 9996$; the quotient is 6784. (*See exam.* 8, *page* 23.)

And if the divisor were 952; half that number set one place to the right, and added, would make 9996.

3229184		476
6458368		952
6781	2864÷	9996
2	7124	
	8	
6783	9996	

Let it be carefully borne in mind that, when a divisor can be simplified, any *multiple, submultiple, or aliquot part* of it, can also be simplified.

Thus, 167 multiplied by 6 gives 1002 for a simple divisor. Now, if we take the multiples, 334, 501, 668, 835, 8350, 1169, 1336, 13360, 1503, 15030, etc., we have: $334 \times 3 = 1002$; $501 \times 2 = 1002$; and by adding to 668 and 835, one half and one fifth of each, respectively, we get 1002. To simplify 1169, 1336 and 1503, we deduct from each, one seventh, one fourth and one third respectively, to get 1002.

And if we take the submultiples, $83\frac{1}{2}$, $55\frac{2}{3}$, $41\frac{3}{4}$, $33\frac{2}{5}$, $27\frac{5}{6}$, $23\frac{6}{7}$, $20\frac{7}{8}$, $18\frac{5}{9}$, etc., we multiply by 2, 3, 4, 5, etc., to get 167, which, in turn, is multiplied by 6, to get 1002, the same simple divisor for all the numbers. (*See page* 70.)

Again, many numbers, which, at first glance, may appear difficult to simplify, will be found, on slight inspection, to be easily managed, and frequently by a choice of several methods. Take for instance, 69, 690, 6900, etc., and their multiples, 138, 1380, etc., 276, 2760, 27600, etc.

EXAM. Divide 50950980 by 690. We give the solution of this in four different ways, as follows:

First: In this, we add to the terms one half of each, respectively; then, setting said half one place to the right under each result and subtracting, we have $738789210 \div 10005$; and the quotient is 73842; the remainder being equal to 1

50950980		690
25475490		345
76426470		1035
25475490		345
73878	9210	÷10005
36	9390	
73841	9820	
	185	
	10005	

Second: Here, we add to the terms a third of each, respectively, and we have 67934640 to be divided by 920. Cutting off the cipher from each, we have $6793464 \div 92$; and the quotient is 73842. (*See exam.* 6, 7 *and* 8, *p.* 23.)

50950980		690
16983660		230
67934	64|0	÷92|0
5434	72	
434	72	
43	72	
2	72	
	40	
73841	92	

NOTE.—It is well to remember that, whether we use the method of addition or that of subtraction, *the figures on the left of the line, only, are to be operated upon;* and if any figure be carried from right to left, over the line, in either case, such figure must also be operated upon. In this last example we multiply 5 (2 plus 3 carried) by 8, to get 40; and in the other example, 37 (36 + 1) is multiplied by 5 to get 185.

Third: Here, we take 700 for approximate divisor; the difference is 10. Cutting off two figures and dividing by 7, divides by 700. We next add $\frac{10}{700}$, or $\frac{1}{70}$ of each partial quotient, till all these quotients are exhausted (divide by 7 and set each result one place to the right) using the fractional form for the remainders. The sum of the several results is 73841 | $89\frac{4}{7}$, which is equal to 73842, the required quotient.

		7..
509509	80 ÷	690
72787	$11\frac{3}{7}$	10
1039	$81\frac{3}{7}$	
14	$84\frac{2}{7}$	
	20	
	$01\frac{3}{7}$	
73841	$98\frac{4}{7}$ = 73842	

Dividing 50950980 by 690, it is scarcely necessary to say, is the same as dividing one seventh of the one by one seventh of the other, or dividing $7278711\frac{3}{7}$ by $98\frac{4}{7}$ ($690 \div 7 = 98\frac{4}{7}$) and, therefore, the remainder $98\frac{4}{7}$ being equal to the divisor, 1 is added making 73842.

Again, in the addition, 1 is carried from the figures on the right, to those on the left of the line, and if written down, the carried figure would be set under 14: the carried figure, therefore, (1 understood) forms a partial quotient the same as the numbers immediately above it; and $\frac{1}{70}$ of this figure, $.01\frac{3}{7}$, must be added.

Now, since $98\frac{4}{7}$ is the simplified divisor, $1\frac{3}{7}$ ($100-98\frac{4}{7}$) is the complement, or multiplier: but multiplying by $1\frac{3}{7}$ is multiplying by $\frac{10}{7}$; and since we have already divided by 100, by means of the line, and now dividing by 7, we have added the $\frac{10}{700}$, or $\frac{1}{70}$, in each case.

Fourth: The process, in this, is the same as in the last example, except that the remainders are treated decimally instead of fractionally.

509509	80
72787	11.428571
1039	81.428571
14	84.285714
	20
	01.428571
73841	98.571428

The remainders, in the present instance, run into circulating decimals, or periodicals, and if carried out, their sum would be .571428, repeated. Now, if $\frac{4}{7}$, the fractional part of $98\frac{4}{7}$, be reduced to a decimal it will be found to be equal to .571428, also, repeated. The divisor $98\frac{4}{7}$, therefore, is equal to 98.571428; and since the remainder is equal to the divisor, 1 is added to the quotient, making 73842, as in the other cases.

From the examples and illustrations given, it will now be readily seen how to divide by 29, 39, 59, 79, 390, 399, 599, 59990, etc.

Again, if we take such numbers as 200, 2000, 300, 30000, 400000, 700, 7000, etc., and deduct the significant figure from these numbers, and also from the successive remainders, each remainder thus found, can be simplified when used for a divisor.

Take for instance, 7000, and deduct 7 from it, and also from each successive remainder, and we find that 6993, 6986, 6979, 6972, 6965, 6958, etc., can be simplified.

If, for example, 6986 were presented for divisor, we would compare it with 7000, as shown in the margin.

Here we see that 14 is the difference between 6986 and 7000, and that 7, the significant figure of 7000, is contained in 14, an exact number of times; and this being understood, 7 will be contained in 6986, also, an exact number of times: and so we divide 6986 by 7 to get 998 for a simple divisor.

```
7. .
6986
----
 998
```

EXAM. Divide 76874626 by 5994, exact, to six places of decimals.

In this, we see at a glance that 6 is the difference between 5994 and 6000; hence 6, the significant figure in 6000, is contained in 5994 giving 999 for a simple divisor. We then divide the dividend by 6, getting 12812437.6' which is divided by 999.

```
76874626                        |6...
--------                        |5994
12812|437.6'   ÷999             |----
   12|812                       | 999
     | 13
-----|----------
12825|262 666|666
          262|666
             |263
     --------|---
      .262929|595
```

In dividing by 6, we treat the remainder decimally, and obtain the repetend .6', in other words, 6 is repeated to infinity, and in getting the decimal part of the quotient, 6 is repeated six times for the six decimal places required. The quotient is 12825.262929, true to six places of decimals.

NOTE. — And if the divisor were 6 less than 5994, that is, 5988, we would divide 5988 by 6, getting 998 for a simple divisor; and so with the other numbers differing by 6. Thus, 5982÷6=997; etc.

When one part of the divisor is a multiple of the other part, which, in itself is a simple divisor, the division can be simplified.

Take, for instance, such numbers as, 96192, 960192, 960120, etc., 93372, 930372, 48144, 480144, 36108, 360108, 24096, 240960, etc.

Suppose, now, it were required to divide by any one of these, say 930372. Arranging the numbers as follows, the component factors will be readily seen:

96	96	93	93
96192	960192	93372	930372
1002	10002	1004	10004

EXAM. Divide 30207318096 by 930372.

A moment's glance, here, shows that 372, one part of the divisor, is 4 times 93, the other part, in other words, 93 and 10004 are component factors of 930372 ($93\times10004=930372$). We now divide, first, by 10004, getting 3019524 for quotient, which in turn, we divide by 93, and the quotient is 32467 | 93=32468.

```
                      93
3020731| 8096÷930372
   1208| 2924   10004
-------|-----
3019523| 5172
       | 4832
       |10004
----------------
   30195 24÷93
    2113 65
     147 91
      10|29
        |84
   -----|--
   32467|93
```

Again, if these numbers be reversed such as, 19296, 192960, etc., the division can also be simplified. To divide by 192960, for instance, we would take half, or 96480, and make use of the component factors 96 and 1005, as shown in the margin, dividing, first, by 1005, and the result found, by 96: or, divide, first, by 96 and next by 1005.

```
   96
96480
 1005
```

And if 48144 were presented for divisor, we would divide by 48, using the factors 6 and 8 ($6\times8=48$); then divide the result by 1003: and so with the others, and similar numbers.

```
   48
48144
 1003
```

Or, doubling 48144 gives 96288, the component factors being 96 and 1003. (*See exam. and note page* 79.)

```
   96
96288
 1003
```

NOTE.—This method of using the component factors, in connection with the simplified methods given in this work, will be of great advantage when the divisor is not subject to change. (*See page* 55, *exam. and note.*)

DECIMAL FRACTIONS, OR DECIMALS.

A Decimal Fraction, or a Decimal, is a fraction having 10, or some power of 10, such as 100, 1000, 10000, etc., for its denominator.

Thus, $\frac{3}{10}$, $\frac{9}{100}$, $\frac{3}{100000}$, $\frac{4756}{1000}$, are decimals.

In the notation of decimals, the denominator is usually omitted, the value of the fraction being expressed by pointing off as many decimal places in the numerator as there are ciphers in the denominator.

Should there not be a sufficient number of figures in the numerator, the deficiency is to be supplied by prefixing ciphers.

Thus, $\frac{3}{10}$, $\frac{9}{100}$, $\frac{3}{100000}$, $\frac{4756}{1000}$, expressed in the notation of decimals, are written, respectively, .3, .09, .00003, 4.756. Hence, conversely:

The denominator of a decimal thus expressed, is unity, or 1, followed by as many ciphers as there are figures in the decimal.

Thus, .57 is $\frac{57}{100}$, .004 is $\frac{4}{1000}$, and .00063 is $\frac{63}{100000}$.

It will be readily seen from this notation, that the figure immediately to the right of the decimal point, is tenths, the next, hundredths, the third thousandths, etc.

Thus, since 376 is equivalent to 300+70+6, the fraction .376, or $\frac{367}{1000}$ is equivalent to $\frac{300}{1000}+\frac{70}{1000}+\frac{6}{1000}$, or $\frac{3}{10}+\frac{7}{100}+\frac{6}{1000}$.

The values of figures in decimals, as in whole numbers, are increased in a tenfold degree by removing the decimal point one place towards the right hand, and are diminished in a like degree by removing the point one place to the left.

Thus, in the decimal .003, by removing the point one place to the right we have .03, which denotes $\frac{3}{100}$, or $\frac{30}{1000}$, and is therefore ten times the given fraction, .003, or $\frac{3}{1000}$; but by removing the point one place to the left we have .0003, or $\frac{3}{10000}$, which is only a tenth part of .003, or $\frac{3}{1000}$, or $\frac{30}{10000}$.

Hence,

A decimal is multiplied by 10, if the point be removed one place

to the right; by 100, if two places; by 1000, if three places, etc., and, conversely, a decimal is divided by 10, if the point be removed one place towards the left hand, by 100, if two places; by 1000 if three places, etc., vacant places, when there are such, being supplied in both cases by ciphers.

Thus, $.6345 \times 10 = 6.345$, or $6\frac{345}{1000}$; $6.345 \times 100 = 634.5$; $6.3 \times 1000 = 6300$. Also, $78.43 \div 10 = 7.843$; $.784 \div 100 = .00784$; $7.3 \div 100 = .073$, etc. Hence,

The value of a decimal is not changed by annexing a cipher to the end of it, nor by taking one away, a cipher not affecting the value of a number except when placed between a significant figure and the decimal point. (See note 2, page 233.)

Thus, $.50 = .5 = .500 = .5000$; each being equivalent to one half.

But if a cipher be placed between the significant figure and the decimal point, the value of the number is changed at once.

Thus, 50., 500., 5000., .05, .005, .0005, etc.

From the foregoing view of the nature of decimals, it will be seen that there is, in every respect, the closest resemblance between them and whole numbers; and hence all operations on decimals are performed exactly in the same manner as those on whole numbers, due attention being paid to the position of the decimal point. This last circumstance, indeed, requires the utmost care when making our calculations, as *the point* is the characteristic of the decimal; and, from what precedes, it is evident how much depends on its proper position.

RULE I. *To reduce a common fraction to a decimal, in other words, to divide a smaller number by a larger: First, annex a cipher to the smaller number and divide by the larger, and to the significant figure or cipher found in the quotient, prefix a point. Then, if there be a remainder, annex ciphers, and continue the division till nothing remains, or till the result consists of as many figures as may be deemed necessary.*

EXAM. Reduce $\frac{15}{2560}$ to a decimal, in other words, divide 15 by 2560, expressing the quotient decimally.

Here, by annexing a cipher 15 becomes 150, in which 2560 is not contained; and therefore a cipher is placed in the quotient. Annexing another cipher, 150 becomes 1500, in which 2560 is not contained, and another cipher is put in the quotient. After this, the division proceeds in the usual way, a cipher being added each time; and the quotient, or decimal is found to be .005859375, or $\frac{5859375}{1000000000}$: which by reduction to its lowest terms would become $\frac{15}{2560}$, the given fraction, thereby proving the work to be correct. (The work is left for the student to perform.)

```
2560)15000(.005859375
      12800
     -------
     etc., etc.
```

NOTE.—It may be well to remark that, when the fractional form of the decimal is used, the point is always omitted, the numerator of the fraction consisting of the *significant* figures of the decimal, while the denominator will always be 1, followed by as many ciphers as there are places in the decimal, including significant figures, and ciphers, if there be any.

RULE II. *To divide decimals: If the number of decimal places in the divisor and dividend be not equal; make them equal by annexing ciphers to the one having the least number. Then reject the points and divide as in whole numbers, and if the divisor be contained in the dividend, the figure, or figures found in the quotient will be a whole number. Having used the last figure of the dividend, annex ciphers, if there be a remainder, and continue the division till nothing remains, or till the number of figures considered necessary is found in the quotient. The part of the quotient thus obtained, will be a decimal.*

If, after rejecting the points, the divisor be greater than the dividend, the work will proceed according to the rule given on page 255.

EXAM. 1. Divide 2738.5 by 78.54.

Here, by annexing a cipher to the dividend, we make the number of decimal places in both numbers equal. Then rejecting the points, we have 273850 to be divided by 7854; and we find the integral part to be 34. Annexing ciphers, and continuing the division we get the decimal part .86758, etc. The quotient, therefore, is 34.86758, etc.

(*For simple method of dividing by* 7854, *see p.* 56.)

EXAM. 2. Divide .1342 by 67.1.

In this, we equalize the number of decimal places in both numbers, by annexing three ciphers to the divisor; and rejecting the points, we have 1342 to be divided by 671000. Then, the divisor being greater than the dividend, we proceed according to rule I, page 255. The required quotient is .002.

MULTIPLICATION OF DECIMALS.

RULE. — *To multiply decimals: Multiply the factors as in simple multiplication and point off in the product as many places of decimals, as there are in both factors; supplying the deficiency, when there is one, by prefixing ciphers.*

EXAM. 1. Multiply 66.3 by .582.

Here, we multiply 663 by 582 as whole numbers; the product is 385866. Then, because there are three decimal places in one factor, and one in the other, there must be four decimal places in the product 38.5866.

EXAM. 2. Multiply .14 by .6.

Here, a cipher must be prefixed to the product 84, as there are two places of decimals in one factor, and one in the other. The product, therefore, is .084.

NOTE. — Recourse should be had to our short methods, whenever possible, in multiplying decimals, as well as whole numbers, as illustrated in the following:

EXAM. 3. Multiply 79.96 by 79.94.

Treating the factors, in this, as whole numbers, 4 and 6 make 10, and the other figures are alike: say 4 times 6 are 24; set down in full; then, add 1 to 799 making 800, and say 800 times 799 are 63920 to complete the product. Pointing off four places now, we have 639.2024. (*See page* 87.)

$$\begin{array}{r} 79.96 \\ 79.94 \\ \hline 639.2024 \end{array}$$

The student's attention is called to the remaining pages of the work, which are devoted exclusively to *short methods*. These methods, so far as known to the author, are entirely original, and will be found both interesting and practical:

TO MULTIPLY THE 'TEENS TOGETHER.

RULE. (1) *Multiply the units by the units and set down the unit figure of the product.* (2) *Add the figure to be carried, if any, to either factor, and to the result; add the unit figure of the other factor, dropping the* 1 *from that other factor, always.*

NOTE.—It is immaterial to which factor the carried figure is added, provided the 1 is dropped from the other, as illustrated in the following:

EXAMPLES:

$17 \times 16 = 272$: 19×18 $16 \times 14\frac{1}{2}$ etc.	Here, say 6 times 7 are 42; set down 2, and carry 4 to 16 are 20, and 7 (in 17) are 27: Or, 4 to 17, are 21, and 6 (in 16) are 27, making 272; the 1 being dropped from the opposite factor in either case.
$18 \times 15\frac{1}{3} = 276$: $17\frac{1}{2} \times 18$ etc.	In this, say $\frac{1}{3}$ of 18 is 6, to carry: then, 5 times 8 are 40, and 6 are 46; set down 6, and carry 4 to 18, are 22, and 5 (in 15) are 27: Or, 4 to 15, are 19, and 8 (in 18) are 27; making 276. The *reason* for

dropping the 1 from either factor will be understood from the following:

$$\begin{array}{r|l} 17 = 10 + & 7 \\ 16 = 10 + & 6 \\ \hline 10 & 0 \\ 6 & 0 \\ 7 & 0 \\ 4 & 2 \\ \hline 27 & 2 \end{array}$$

If the *parts* of the numbers be multiplied together, instead of the numbers themselves, as shown in the margin, we have, first, $10 \times 10 = 100$; then $10 \times 6 = 60$; next, $7 \times 10 = 70$ and finally, $7 \times 6 = 42$. Adding the several products thus found, we get 272, or 16 times 17. And here, it will be observed that, having set down 2, the unit figure in 42; we add, or carry 4 to 6 and 10 (16) leaving 7 (the 1 being dropped) to be still added. Or, carrying 4 to 7 and 10 (17) leaves 6 (the 1 being dropped) to be added.

And when the 'teens are taken in a reversed order,

12, 13, 14, 15, 16, 17, 18 and 19 will become:

21, 31, 41, 51, 61, 71, 81 and 91, each ending in 1.

Now,

Any two figures ending in 1 can be multiplied together by the following simple rule: (1) *Set down the unit figure.* (2) *Add the tens and set down the unit figure of the sum.* (3) *Multiply the tens together, carrying as usual;* thus:

$91 \times 81 = 7371$:	In this, set down the unit figure 1; then add: 8 and 9 are 17; set down 7, and carry 1: now multiply; 8 times 9 are 72, and 1 are 73 completes the product.
71×61	
31×71	
etc.	

The *reason* for adding in the second part of the process will be understood if the numbers be multiplied together, in the usual way, as shown in the margin. Here, we see that, having set down the unit figure, the figures 8 and 9 are repeated in the work, and added, which can be done without setting them down a second time.

$$\begin{array}{r} 91 \\ \underline{81} \\ 91 \\ \underline{728} \\ 7371 \end{array}$$

TO MULTIPLY THE TWENTIES, THIRTIES, FORTIES, ETC., TOGETHER.

The Twenties, Thirties, Forties, Fifties, Sixties, Seventies, Eighties and Nineties, when written in the natural order, can be multiplied together by the following:

RULE. (1) *Multiply the units by the units.* (2) *Multiply the sum of the units by a single figure of the tens.* (3) *Multiply the tens by the tens; carrying as usual;* thus:

$23 \times 24 = 552$:	Here, we say 4 times 3 are 12; set down 2, and carry 1: then, 2 times 7 $(4 + 3)$ or, 7 times 2, are 14, and 1 are 15; 5, and carry 1: next, 2 times 2 are 4, and 1 are 5 completes the product.
25×27	
27×29	
28×28	
etc.	

The factors in these examples, are written in their natural order, and here it may be remarked that, when thus written, the *tens* in each set of numbers are alike, and the *reason* of the rule will be understood from the following:

EXAM. Multiply 73 by 72, as shown in the margin:

Here, having set down 2 times 3, we have next, 2 times 7; then, 7 times 3, or 3 times 7; that is, 5 (2 + 3) times 7; and finally, 7 times 7, plus the figure carried.

$$\begin{array}{r} 73 \\ \underline{72} \\ 5256 \end{array}$$

$37 \times 38 = 1406$: Say 8 times 7 are 56; 6 and carry 5: now 3 times
32×36 15 (8 + 7) are 45, and 5 are 50; 0, and carry 5:
etc. then, 3 times 3 are 9, and 5 are 14; making 1406.

43×48 It will be seen that the rule is applicable to all the
42×45 numbers down the margin; *but in all cases where the*
etc. *unit figures equal* 10, *while the tens are alike*, the process
62×69 can be still further simplified by the following Rule:
67×64 (1) *Multiply the units together and set down the product*
etc. *in full.* (2) *Add* 1 *to either of the tens and multiply the other by the figure thus increased;* as:

$76 \times 74 = 5624$: In this, the units, 4 and 6, make 10, and the 7's
78×72 are alike. Say 4 times 6 are 24; set down in full;
etc. now, add 1 to 7, and say 8 times 7 are 56 to complete the product.

$81 \times 89 = 7209$: When the unit figures are 1 and 9 they give
61×69 only one figure when multiplied, and in such
etc. cases a cipher is set in the second place, as $81 \times 89 = 7209$. (*See page* 87, *and note.*)

The rule is applicable to numbers of three figures, also; and in many cases, to four, or more figures; thus:

$126 \times 124 = 15624$: Say 4 times 6 are 24; set down in full; add
123×127 1 to 12 and say 12 times 13 are 156; making 15624.

$398 \times 392 = 156016$: Say 2 times 8 are 16; add 1 to 39, and say 40
493×497 times 39 are 1560; making 156016.

$7994 \times 7996 = 63920024$: Say 6 times 4 are 24; set down in full;
3998×3992 then, add 1 to 799 and $799 \times 800 = 639200$
etc. completes the product.

And the rule can be extended to all numbers of a like nature.

Having thus shown how the twenties, thirties, etc., can be multiplied together when written in the natural order, let us take them, now, in a reversed order: Thus, reversing

23, 24; 32, 37; 48, 49; 57, 58; 73, 76; 87, 89; 92, 98, etc., we have

32, 42; 23, 73; 84, 94; 75, 85; 37, 67; 78, 98; 29, 89, etc., and here it will be observed that the *units*, in each set of numbers, are alike.

Now,

To multiply any two figures by any other two, whose units are alike, we give the following Rule: (1) *Multiply the units by the units.* (2) *Multiply the sum of the tens by a single figure of the units.* (3) *Multiply the tens by the tens; carrying as usual;* thus:

$32 \times 42 = 1344$:	Say 2 times 2 are 4; set down 4; then, 7 $(4 + 3)$ times 2 are 14; 4 and carry 1: next, 4 times 3 are 12, and 1 are 13 completes the product.
53×43	
37×67	

The *reason* is plain: Take 74×64, as shown in the margin.

$74 \times 64 = 4736$:	Having multiplied the units together, and set down 6, we have 4 times 7; then, 6 times 4, or 4 times 6; that is, 4 times 13 $(6 + 7)$ plus the 1 carried; then 6 times 7, plus 5.	$\begin{array}{r} 74 \\ \underline{64} \\ 4736 \end{array}$
83×93		
94×74		
etc.		

And the rule can be applied with equal facility to numbers of *three figures, whose tens and hundreds consist of the 'teens and whose units are alike;* thus:

$174 \times 164 = 28536$:	In this, we say 4 times 4 are 16, 6, and carry 1: then, 4 times 33 $(16+17)$ are 132, and 1 are 133; set down 3, and carry 13: finally 16 times 17 (*short method, page* 258) are 272, and 13 are 285, which completes the product.
163×183	
148×158	
123×133	
156×156	

NOTE.—It will be observed, on examination, that the method made use of in multiplying 174 by 164, is the same as that in multiplying 74 by 64.

And if the three figures be such that the hundreds in both factors are alike, while the units and tens consist of the 'teens; as, 417, 416, 318, 319, etc., we have the following

RULE. *To multiply any two numbers of three figures together, whose hundreds are alike, and whose units and tens consist of the 'teens:* (1) *Multiply the 'teens together and set down the units and tens of the product, carrying the hundreds.* (2) *Multiply the sum of the 'teens by a single figure of the hundreds; set down the units and tens of the product, and carry the hundreds.* (3) *Multiply the hundreds together to complete the product;* thus:

$417 \times 416 = 173472$:	In this, 16 times 17 are 272 (*short method*); set down 72, and carry 2 hundred: then 4 times 33 $(16 + 17)$ are 132, and 2 are 134; set down 34, and carry 1: next, 4 times 4 are 16, and 1 are 17 completes the product.
115×114	
817×813	
215×215	
etc.	

Should the tens in these numbers be a cipher, instead of 1, *proceed as follows:* (1) *Multiply the units together and set down the product in full.* (2) *Multiply one figure of the hundreds by the sum of the units, setting down the product, also, in full.* (3) *Multiply the hundreds together and set down the product in full;* thus:

$804 \times 803 = 645612$:	Here, we say 3 times 4 are 12; set down in full; then, 7 $(3 + 4)$ times 8 are 56; set down, also, in full; and 8 times 8 are 64 completes the product.
602×608	
705×705	
etc.	

In multiplying numbers of this nature together, it is well to remember that *two figures are always set down for the product of the units; and two, also, for the product of the hundreds by the sum of the units.* When the product of the units is only one figure, a cipher is put in the second place; and when the product of the hundreds by the sum of the units consists of three figures, two only, are set down, the third, or hundreds being carried to the next product; illustrated in the following:

$803 \times 803 = 644809$:	Here, 3 times 3 are 9 (one figure) set down two, 09; then, 6 $(3 + 3)$ times 8 are 48; set down in full; next, 8 times 8 are 64 completes the work.
704×702	
801×809	
etc.	
$906 \times 907 = 821742$:	Say 7 times 6 are 42; set down in full; then, 13 $(6 + 7)$ times 9, or 9 times 13 are 117 (three figures); set down two, 17, and carry 1 hundred; finally, 9 times 9 are 81, and 1 are 82.
904×908	
807×807	
709×708	

The rule can be applied with equal facility to *numbers of four figures, also; when the tens in both numbers are ciphers and the third and fourth figures consist of like 'teens;* thus:

$1103 \times 1106 = 1219918$:	In this, say 6 times 3 are 18, and set down in full; then, 9 $(3 + 6)$ times 11 are 99; set down in full; next, 11 times 11 are 121 completes the work.
1204×1203	
1305×1304	
etc.	

Or, commencing with the left hand figures: we have 11 times 11 are 121; set down in full; then 11 times 9 (3 + 6) are 99; set down in full; and 3 times 6 are 18 completes the work as before.

It may be proper to remark here, also, that in multiplying numbers of this nature together, *two figures, or a significant figure and a cipher to make two*, are always set down for the product of the units. And *two figures only*, are set down for the product of the 'teens by the sum of the units. Should the product give three figures, the units and tens are set down, and the hundreds are carried to the next product; thus:

$1903 \times 1903 = 3621409$:	Here, 3 times 3 are 9 (one figure) set down two, 09; then 6 (3 + 3) times 19 are 114 (three figures); set down two, 14, and carry 1 hundred: finally, 19 times 19 (*short method*) are 361, and 1 are 362 completes the product.
1804×1802	
1701×1709	
1608×1601	
etc.	

And when the foregoing numbers are *reversed* they can be multiplied together with equal facility; thus: 906 and 907 become 609 and 709, the units in both numbers being alike, while the hundreds are unlike:

$609 \times 709 = 431781$:	Here, 9 times 9 are 81; set down in full; then, 9 times 13 (6 + 7) are 117 (three figures); set down two, 17, and carry 1 hundred; next, 7 times 6 are 42, and 1 are 43 completes the product.
907×807	
406×706	
etc.	

And any *two numbers of four figures* whose units and tens consist of the 'teens, the third figure being a cipher, while the fourth in each is alike, can be multiplied together by the following:

RULE. (1) *Multiply the 'teens together (short method) and set down the product in full.* (2) *Multiply the sum of the 'teens by one figure of the thousands and set down the product, also, in full.* (3) *Multiply the thousands together to complete the product;* thus:

$4017 \times 4016 = 16132272$:	In this, say 16 times 17 are 272; set down in full; then, 4 times 33 (16 + 17) are 132; set down, also, in full; next, 4 times 4 are 16 completes the product.
3014×3018	
5015×5015	
etc.	

Or, commencing with the left hand figures, say 4 times 4 are 16; set down in full; then, 4 times 33 are 132; set down in full; and 16 times 17 are 272 completes the product, as before.

If, when the sum of the 'teens is multiplied by one figure of the thousands, the product gives only two figures, then a cipher is set down to make three places, always; thus:

$2019 \times 2019 = 4076361$:
3014×3016
3011×3015
etc.

Here, 19 times 19 are 361; set down in full; then 2 times 38 $(19 + 19)$ are 76 (two figures), set down three, 076; next, 2 times 2 are 4 completes the product.

From the foregoing examples and illustrations, the student will find no difficulty, now, in extending the rules to the twenties, thirties, forties, etc., thus:

To multiply any two numbers of three figures together when the hundreds are alike, and whose units and tens consist of the twenties, thirties, etc.:

RULE. (1) *Multiply the twenties, thirties, etc., together and set down the two first figures of the product, always, carrying the others.* (2) *Multiply the sum of the twenties, etc., by a single figure of the hundreds and set down two figures, only, of the product.* (3) *Multiply the hundreds together, adding the carried figures to complete the product.*

EXAM. Multiply 823 by 824.

In this, we say 24 times 23 are 552 (*short method, page* 259) set down 52, and carry 5 hundred: then, 8 times 47 $(23 + 24)$ are 376, and 5 are 381; set down 81, and carry 3 hundred: next, 8 times 8 are 64, and 3 are 67 completes the product.

$$\begin{array}{r|l} 8 & 23 \\ 8 & 24 \\ \hline \multicolumn{2}{l}{678152} \\ \multicolumn{2}{l}{\ \ {\scriptstyle 3\ \ 5}} \end{array} \left.\vphantom{\begin{array}{c}8\\8\end{array}}\right\} 47$$

And if ciphers come between the hundreds and the twenties, thirties, etc., the process will be still more simple; thus:

EXAM. Multiply 8023 by 8024.

Here, we have $23 \times 24 = 552$ (*short method*) which is set down in full; then, 8 times 47 $(23 + 24)$ are 376, which is set down, also, in full; next, 8 times 8 are 64 completes the product.

$$\begin{array}{r} 8023 \\ 8024 \\ \hline 64376552 \end{array}$$

EXAM. Multiply 80023 by 80024.

In this, we have $23 \times 24 = 552$; set down in full; then a cipher: now, 8 times 47 are 376; set down in full; then a cipher: next, 8 times 8 are 64 completes the product.

$$\begin{array}{r} 80023 \\ 80024 \\ \hline 6403760552 \end{array}$$

EXAM. Multiply 8094 by 8093.

In this, we have 93 times 94 (*short method*) $= 8742$; set down 742, and carry 8: then, 8 times 187 are 1496, and 8 are 1504; set down 504, and carry 1: next, 8 times 8 are 64, and 1 are 65 completes the product.

$$\begin{array}{r} \left.\begin{array}{r|l} 80 & 94 \\ 80 & 93 \end{array}\right\} 187 \\ \hline 65504742 \\ {\scriptstyle 1 \quad\;\; 8} \end{array}$$

EXAM. Multiply 80094 by 80093.

Here, $94 \times 93 = 8742$; thus: 3 times 4 are 12; 2 and carry 1: then, 7 $(3 + 4)$ times 9, or 9 times 7 are 63, and 1 are 64; 4 and carry 6: next, 9 times 9 are 81, and 6 are 87; set down 8742 in full; now, 8 times 187 are 1496; set down, also, in full; finally, 8 times 8 are 64 completes the product.

$$\begin{array}{r} \left.\begin{array}{r|l} 800 & 94 \\ 800 & 93 \end{array}\right\} 187 \\ \hline 6414968742 \end{array}$$

EXAM. Multiply 1034 by 1038.

In this, 38 times 34 are 1292 (*short method*); four figures, set down only three, 292, and carry 1: then, 72×1 plus 1 carried are 73 two figures, set down three, 073; next, 1 multiplied by 1 completes the product.

$$\begin{array}{r} \left.\begin{array}{r|l} 10 & 34 \\ 10 & 38 \end{array}\right\} 72 \\ \hline 1073292 \end{array}$$

REMARK. — In multiplying numbers of this nature together, it will be observed that, when the factors consist of three figures, only two are set down for the product of the units and tens; and two figures, also, for the product of their sum by the hundreds. When one cipher intervenes, three figures are set down in each case, and when two ciphers intervene, four figures, and so on, according to the number of ciphers. When less than the required number of figures in the product, a cipher is set down, as 073, in the last example.

EXAM. Multiply 160042 by 160048.

Here, $42 \times 48 = 2016$; set down in full; then, 16 times 90 are 1440; and 16 times 16 are 256 completes the product.

$$\begin{array}{l} \left.\begin{array}{r|l} 1600 & 42 \\ 1600 & 48 \end{array}\right\} 90 \\ \hline 25614402016 \end{array}$$

The process will be found equally simple *in cases of three figures where the hundreds are not alike, the units and tens in both factors consisting of any figures;* and, also, under similar conditions, when ciphers intervene; thus:

EXAM. Multiply 518 by 314.

In this, 14 times 18 (*short method*) are 252 (three figures); set down two, only, 52, and carry 2: then, 3 times 18 are 54, and 2 are 56 and 5 times 14 are 70; now, 70 and 56 are 126; set down 26, and carry 1: next, 3 times 5 are 15, and 1 are 16 completes the product.

$$\begin{array}{l} \left.\begin{array}{r|l} 5 & 18 \\ 3 & 14 \end{array}\right\} \begin{array}{r} 56 \\ 70 \end{array} \\ \hline 162652 \end{array}$$

EXAM. Multiply 518 by 304.

Here, 4 times 18 are 72; set down in full; then, 3 times 18 are 54 and 5 times 4 are 20; the sum of both is 74; set down in full; now, 3 times 5 are 15 completes the product.

$$\begin{array}{l} \left.\begin{array}{r|l} 5 & 18 \\ 3 & 04 \end{array}\right\} \begin{array}{r} 54 \\ 20 \end{array} \\ \hline 157472 \end{array}$$

EXAM. Multiply 473 by 604.

In this, 4 times 73 are 292 (three figures); set down two, 92, and carry 2: then, 6 times 73 are 438, and 2 are 440 and 4 times 4 are 16; the sum of both is 456; set down 56, and carry 4: now, 6 times 4 are 24, and 4 are 28 completes the product.

$$\begin{array}{l} \left.\begin{array}{r|l} 4 & 73 \\ 6 & 04 \end{array}\right\} \begin{array}{r} 440 \\ 16 \end{array} \\ \hline 285692 \end{array}$$

EXAM. Multiply 4073 by 604.

Here, 4 times 73 are 292; set down 92, and carry 2: then, 6 times 73 are 438, and 2 are 440 and 4 times 40 are 160; the sum of both is 600; set down in full; next, 6 times 4 are 24 completes the product.

$$\begin{array}{l} \left.\begin{array}{r|l} 40 & 73 \\ 6 & 04 \end{array}\right\} \begin{array}{r} 440 \\ 160 \end{array} \\ \hline 2460092 \end{array}$$

EXAM. Multiply 7034 by 6023.

In this, 23 times 34 are 782; set down in full; then, 6 times 34 are 204 and 7 times 23 are 161; the sum of both is 365; set down in full; now, 6 times 7 are 42 completes the product.

70	34	}	204
60	23	}	161
42365782			

Before proceeding with our examples, it will be of advantage to the student to become *thoroughly familiar* with the following method:

To multiply any two digits by any other two, in a single line; illustrated in the following:

EXAM. Multiply 73 by 82.

In this, say 2 times 73 are 146; set down 6, and carry 14: then, 8 times 3 are 24, and 14 are 38; 8 and carry 3: now, 8 times 7 are 56, and 3 are 59 completes the product.

73	146
82	
5986	

EXAM. Multiply 67 by 42.

Here, 2 times 67 are 134; set down 4, and carry 13: then, 4 times 7 are 28, and 13 are 41; 1 and carry 4: now, 4 times 6 are 24, and 4 are 28 completes the product.

$$\begin{array}{r} 67 \\ 42 \\ \hline 2814 \end{array}$$

NOTE.—When the student has become familiar with the process, it will not be necessary to set down the product in the margin, as shown in the first example.

And when ciphers intervene, proceed as follows:

EXAM. Multiply 607 by 402.

In this, 2 times 7 are 14; set down in full; then, multiply crosswise: 2 times 6 are 12 and 4 times 7 are 28; now add both products; 28 and 12 are 40; set down in full; and 4 times 6 are 24 completes the product.

$$\begin{array}{r} 607 \\ 402 \\ \hline 244014 \end{array}$$

EXAM. Multiply 7003 by 4006.

Here, 6 times 3, or 18, is set down, then a cipher to make three places; now, 6 times 7 are 42 and 4 times 3 are 12; 42 and 12 are 54; set down, then a cipher to make three places, also; now, 4 times 7 are 28 completes the product.

$$\begin{array}{r} 7003 \\ 4006 \\ \hline 28054018 \end{array}$$

REMARK.—In multiplying numbers of this nature together, it will be observed that, when only one cipher intervenes, *two figures* are set down for the product of the units; and two figures, also, for the result found in multiplying crosswise; and when two ciphers intervene, *three figures* are set down in both cases, and so on, according to the number of ciphers. When the products are less, a cipher, or ciphers are set down to make the required number, as in the foregoing example; illustrated also in the following:

EXAM. 702 by 504.

$$\begin{array}{r}
\left.\begin{array}{r}702\\504\end{array}\right\}\begin{array}{r}10\\28\end{array}\\
\hline
353808
\end{array}$$

In this, set down 4 times 2, then a cipher, 08, to make two places; then, 4 times 7 are 28 and 5 times 2 are 10; 28 and 10 are 38; set down in full; and 5 times 7 are 35 completes the product.

And any number of three digits can be multiplied by any other three, in a single line as follows:

EXAM. Multiply 724 by 348.

$$\begin{array}{r}
\left.\begin{array}{r|l}7&24\\3&48\end{array}\right\}\begin{array}{r}83\\336\end{array}\\
\hline
251952
\end{array}$$

Here, by the method for multiplying two digits by any other two, given on the preceding page, we have $24 \times 48 = 1152$; set down 52, and carry 11: then, 3 times 24 are 72, and 11 are 83 and 7 times 48 are 336, both set on the margin to the right; now, the sum of both numbers is 419; set down 19, and carry 4: next, 3 times 7 are 21, and 4 are 25 completes the product.

If ciphers intervene proceed as follows:

EXAM. Multiply 7024 by 3048.

$$\begin{array}{r}
\left.\begin{array}{r|l}70&24\\30&48\end{array}\right\}\begin{array}{r}73\\336\end{array}\\
\hline
21409152
\end{array}$$

In this, $24 \times 48 = 1152$: set down three figures, 152, and carry 1: then, 3 times 24 are 72, and 1 are 73 and 7 times 48 are 336; their sum is 409; set down in full; next, 3 times 7 are 21 completes the product.

GENERAL SHORT METHOD.

The following short method can be applied in all cases:

Exam. 1. Multiply 78 by 63.

When the multiplier consists of two figures:

Here, say 3 times 8 are 24; set down 4 for the first figure of the answer *one* place to the right of the multiplicand; carry 2, then, 3 times 7 are 21, and 2 are 23; set under the units and tens of the multiplicand, as shown: next, 6 times 8 are 48, and 3 (*add downwards*) are 51; set down 1 and carry 5; 6 times 7 are 42, and 5 are 47, and 2 (*adding downwards*) are 49 completes the product.

```
78×63
23
4914
```

Exam. 2. Multiply 3478 by 74.

In this, 4 times 3478 are 13912; set down 2 for the first figure of the answer, and 1391 under the multiplicand as shown: then, 7 times 8 are 56, and 1 are 57; set down 7 and carry 5; 7 times 7 are 49, and 5 are 54, and 9 (*add downwards*) are 63. Proceeding, we obtain 257372, the product.

```
3478×74
1391
257372
```

Exam. 3. Multiply 2356 by 347.

When the multiplier consists of three figures:

Here, we set 2, the first figure of the answer, *two* places to the right, and 1649 is set as shown in the margin: next, 4 times 6 are 24, and 9 are 33, which gives the second figure of the answer, and 3 to carry; then 4 times 235 plus 3, gives 943, which is set in proper position, making the second partial product 9433, the unit figure 3 being the second of the answer. Now, 3 times 6 are 18, and 7 (4+3 *adding downwards*) are 25; set down 5 and carry 2; 3 times 5 are 15, and 2 are 17, and 10 (6+4 *downwards*) are 27. Proceeding, we obtain the product, 817532.

```
2356×347
 1649
 943
817532
```

Exam. 4. Multiply 15673 by 5432.

When the multiplier consists of four figures:

15673×5432
3134
4702
6269
85135736

In this, 6, the first figure of the answer is set *three* places to the right, and the remaining part of the product, 3134, is set as shown: next, 3 times 3 are 9, and 4 are 13, gives 3 for the second figure of the answer and 1 to carry; and 4702 set down as shown: then, 4 times 3 are 12, and 5 (3+2) are 17, gives 7 for the third figure of the answer, and 1 to carry; and 6269 is set in proper position. Finally 5 times 3 are 15, and 10 (1+0+9) are 25, this gives 5, the fourth figure of the answer, and 2 to carry. Proceeding thus, we obtain 85135736, the complete product.

Exam. 5. Multiply 2346 by 3204.

2346×3204
938
469
7516584

Here, the product by 4 is 9384; 4 is set *three* places to the right, for the first figure of the answer, and 938 as shown: then, the next figure of the multiplier being a cipher, the 8 is brought down for the second figure of the answer. Now, 2 times 6 are 12, and 3 are 15, gives 5 for the third figure, and 469 is set in proper position as shown. Finally 3 times 6 are 18, and 9+9 are 36, gives the fourth figure of the answer, and 3 to carry. Proceeding, we obtain 7516584, the complete product.

NOTE.—It will be observed that the unit figure of the answer is set to the right of the multiplicand *as many places as there are figures in the multiplier less one*, and that the partial products are one less than the *significant figures* of the multiplier, always.

Exam. 6. What is the cost of 347 lbs. of sugar, @ $5.32 per hundred?

USUAL METHOD.
347
5.32
694
1041
1735
$18.4604

SHORT METHOD.
347×5.32
69
105
$18.4604

SQUARING OF NUMBERS.

EXAM. What is the square of 73?

In this, say 3 times 3 are 9; then, 6 (3 + 3) times 7, or 7 times 6 are 42; 2, and carry 4: now, 7 times 7 are 49, and 4 are 53 complete the square. (*See pages* 259, 260 *and* 261.)

$$\begin{array}{l} \left.\begin{array}{r} 73 \\ 73 \end{array}\right\} 6 \\ \hline 5329 \end{array}$$

EXAM. What is the square of 417?

Here, the units and tens consist of the 'teens and the hundreds are alike: 17 times 17 (*short method, page* 258) are 289; set down 89, and carry 2: then, 4 times 34 (17 + 17) are 136, and 2 are 138; set down 38, and carry 1: now, 4 times 4 are 16, and 1 are 17; this completes the square.

$$\begin{array}{l} \left.\begin{array}{r|l} 4 & 17 \\ 4 & 17 \end{array}\right\} 34 \\ \hline 173889 \end{array}$$

EXAM. What is the square of 824?

In this, the units and tens consist of the twenties, and the hundreds are alike: 24 times 24 (*short method, page* 259) are 576; set down 76, and carry 5: then, 8 times 48 are 384, and 5 are 389; set down 89, and carry 3: now, 8 times 8 are 64, and 3 are 67 completes the square.

$$\begin{array}{l} \left.\begin{array}{r|l} 8 & 24 \\ 8 & 24 \end{array}\right\} 48 \\ \hline 678976 \end{array}$$

REMARK.—It may be well to remark, here, that, in squaring numbers of three digits, only *two figures* are set down for the product of the units and tens; and *two*, also, for the product of their sum by the hundreds, the remaining figures being carried in both cases.

EXAM. What is the square of 1236?

Here, $36 \times 36 = 1296$; set down 96, and carry 12: then, 12 times 72 are 864, and 12 are 876; set down 76, and carry 8: now, 12 times 12 are 144, and 8 are 152 completes the square.

$$\begin{array}{l} \left.\begin{array}{r|l} 12 & 36 \\ 12 & 36 \end{array}\right\} 72 \\ \hline 1527696 \end{array}$$

When a cipher intervenes any number of three figures can be squared at sight; thus:

EXAM. What is the square of 808 ?

In this, say 8 times 8 are 64; set down in full; then, double 64; 2 times 64 are 128; set down 28, and carry 1: now, 8 times 8 are 64, and 1 are 65 completes the square.

$$\begin{array}{r} 808 \\ 808 \\ \hline 652864 \end{array}$$

EXAM. What is the square of 12034 ?

Here, the units and tens consist of the thirties, and we have $34 \times 34 = 1156$ (*short method*); set nown 156, and carry 1: then, 12 times 68 are 816, and 1 are 817 (three figures) set down in full; now, 12 times 12 are 144 completes the square. (*See pages* 265 *and* 266.)

$$\begin{array}{l} \left.\begin{array}{r|l} 120 & 34 \\ 120 & 34 \end{array}\right\} 68 \\ \hline 144817156 \end{array}$$

EXAM. What is the square of 12734 ?

In this, we first multiply as if the 7's were ciphers, that is, 12034×12034, as in the previous example. To this we have to add 7 times 68 $(34 + 34)$; 7 times 7 and 7 times 24 $(12 + 12)$; that is, 700 times, 7 being in the hundreds' place; thus: 7 times 8 are 56; 6 and carry 5: 7 times 6 are 42, and 5 are 47; 7 and carry 4: 7 times 7 are 49, and 4 are 53; 3, and carry 5: now, 7 times 24 are 168, and 5 are 173; the sum of both products is the square. (*See page* 270.)

$$\begin{array}{r} \left.\begin{array}{r|r|l} 12 & 7 & 34 \\ 12 & 7 & 34 \end{array}\right\} 68 \\ \hline 144817156 \\ 173376. \\ \hline 162154756 \end{array}$$

EXAM. What is the square of 70342 ?

Here, $42 \times 42 = 1764$; set down in full; then, 7 times 84 are 588 (only three figures); set down four, 0588; now, 7 times 7 are 49; the result is the product of 70042 by 70042, to which is added 3 (that is, 300) times 3 | 84, or 1152 and 3 times 14 $(7 + 7)$ or 6 $(3 + 3)$ times 7 are 42; the sum of both products is the square.

$$\begin{array}{r} \left.\begin{array}{r|l} 703 & 42 \\ 703 & 42 \end{array}\right\} 84 \\ \hline 4905881764 \\ 421152.. \\ \hline 4947996964 \end{array}$$

The following will be found interesting and practical:

Any number of three figures can be multiplied by any other number of three figures when the hundreds in both are alike, and whose units and tens, when added, make 10, 20, 30, 40, 50, 60, etc., 100, 110, 120, 130, 140, etc., as follows:

EXAM. Multiply 742 by 708.

In this, 42 and 8 make 50, and the hundreds are alike: Omit the cipher in 50, and set 5 above the 4: now, say 8 times 42 are 336; set down in full; then 7 times 75 are 525 completes the product.

$$\begin{array}{r|l} & 5 \\ 7 & 42 \\ 7 & 08 \\ \hline \multicolumn{2}{r}{525336} \end{array}$$

EXAM. Multiply 518 by 512.

Here, 12 and 18 make 30, and the hundreds are alike: Say 12 times 18 are 216; and 5 times 53 are 265.

$$\begin{array}{r|l} & 3 \\ 5 & 18 \\ 5 & 12 \\ \hline \multicolumn{2}{r}{265216} \end{array}$$

REMARK.—In multiplying numbers of this nature together, *three figures* must be always set down for the product of the units and tens; when four figures are obtained, the fourth is carried; and when only two are obtained, a cipher is set down in the third place; illustrated in the following:

EXAM. Multiply 439 by 471.

In this, 71 and 39 make 110; set 11 above 43: now, by the short method, page 267; we have $39 \times 71 = 2769$; set down 769, and carry 2: then, 4 times 51 are 204, and 2 are 206.

$$\begin{array}{r|l} 1 & 1 \\ 4 & 39 \\ 4 & 71 \\ \hline \multicolumn{2}{r}{206769} \end{array}$$

EXAM. Multiply 748 by 702.

Here, 48 and 2 make 50; set 5 above the 4: now, 2 times 48 are 96 (only two figures); set down three, 096; then, 7 times 75 are 525 completes the product.

$$\begin{array}{r|l} & 5 \\ 7 & 48 \\ 7 & 02 \\ \hline \multicolumn{2}{r}{525096} \end{array}$$

And *when a cipher intervenes in both factors*, the process will be found equally simple; but in this case *four* figures are set down for the product of the units and tens: if only three are obtained, a cipher is set in the fourth place; illustrated in the following:

EXAM. Multiply 3071 by 3089.

In this, 89 and 71 make 160; set 16 above 07: now, $71 \times 89 = 6319$ (*short method*); set down in full; then, 3 times 316 are 948 completes the product.

$$\begin{array}{r|l} 1 & 6 \\ 30 & 71 \\ 30 & 89 \\ \hline \multicolumn{2}{r}{9486319} \end{array}$$

EXAM. Multiply 7024 by 7016.

Here, 16 and 24 make 40; set 4 above the 2. now, 16 times 24 are 384 (three figures) set down four, 0384; then, 7 times 704 are 4928 completes the product. Or, having set down 384, say 70 times 704 are 49280.

```
    |4
  70|24
  70|16
49280384
```

When the units and tens make an even 100; set down their product and make *four* figures always, setting down a cipher or ciphers when four are not obtained; then add 1 to either figure of the hundreds, and multiply by the other; thus:

EXAM. Multiply 988 by 912.

In this, 88 and 12 make 100: say 12 times 88 are 1056; set down in full; now, add 1 to either 9, and say 10 times 9 are 90 to complete the product.

```
  9|88
  9|12
901056
```

EXAM. Multiply 792 by 708.

Here, 92 and 8 make 100: say 8 times 92 are 736; set this down, then a cipher to make four places; now, 8 times 7 are 56 completes the product.

```
  7 92
  7|08
560736
```

EXAM. Multiply 1299 by 1201.

In this, 99 times 1 are 99; set this down, then two ciphers; now, 12 times 13 are 156 completes the product.

```
  12|99
  12|01
1560099
```

EXAM. Multiply 5938 by 5962.

Here, 62 and 38 make 100, and the hundreds and thousands in both factors are alike: now, 62 times 38 (*short method*) are 2356; set down in full; then, adding 1 to either 59. say 60 times 59 are 3540 to complete the product.

```
  59|38
  59|62
35402356
```

And if a figure be changed in either factor, the product is obtained with equal facility; thus:

EXAM. Multiply 5938 by 5964.

In this, we multiply as if 64 were 62, as in the preceding example, and to the result thus found, 2 times 5938, or 11876, is added for the required result.

59	38
59	64
35402356	
11876	
35414232	

EXAM. Multiply 5938 by 5762.

Here, we assume 57 to be 59, and multiply as in the preceding examples, and from the result, 2 (that is, 200) times 5938, or 1187600, is deducted; the difference is the required product. And if the multiplier were 6162 instead of 5762, we would add 1187600.

59	38
57	62
35402356	
11876..	
34214756	

EXAM. Multiply 15947 by 8953.

We multiply in this as if 159 were 89, and we have $8947 \times 8953 = 80102491$, to which is added 7 (that is, 7000) times 8953, or 62671000, to get.................

159	47
89	53
80102491	
62671...	
142773491	

A knowledge of the foregoing methods, with a little practice, will now enable us to obtain extraordinary results, without much mental effort, in many cases where, by the usual methods, it would require considerable labor. Take, for instance, the following:

EXAM. Multiply 120342 by 120358.

1203	42
1203	58
14484122436	

In this, 58 and 42 make 100, and the remaining figures are alike: $42 \times 58 = 2436$ (*short method, page* 267); set down in full; now, adding 1 to 3 (either one); we have 1204×1203: say 3 times 4 are 12; set down in full; next, 7 $(3 + 4)$ times 12 are 84; set down in full; and 12 times 12 are 144; set down in full, completes the product. (*See page* 262.)

EXAM. Multiply 82341 by 2359.

Here, we have, first, 8 (that is, 80000) times 2359, or 18872, set five places to the left of units; then, $2341 \times 2359 = 5522419$, which is set in proper position, and added; this gives the required product $41 \times 59 = 2419$; then adding 1 to 23, we have $24 \times 23 = 552$.

```
 823|41
  23|59
 ---------
 18872....
   5522419
 ---------
 194242419
```

7854

The following method for multiplying any number by 7854, or .7854, so much used in practical mathematics, will be found preferable to the usual method: multiply 34628 by .7854.

First, multiply by 7 and set down the result a second time, one place farther to the right; then, double the latter number and set down the result a second time, also, each one place to the right, and add the results. The *reason* is shown on the margin.

```
34628 × .7854                 ⎧ 7000
-------------                 ⎪  700
242396                 7854 = ⎨  140
 242396                       ⎩   14
  484792                      ------
   484792                       7854
-------------
27196.8312
```

Numbers ending in 5, when not too large, can be readily multiplied together, or squared, by the following:

RULE. (1) *Multiply the units together and set down the product in full.* (2) *Multiply the remaining figures together and to their product add half their sum;* thus:

$165 \times 45 = 7425$:	In this, say 5 times 5 are 25; set down in full;
245×65	then, 4 times 16 are 64, and 8 (half of 16) are 72,
145×85	and 2 (half of 4) are 74 completes the product.
etc.	When the sum of the numbers is odd, drop 1 to
	make the number even, then add half this even

number, and set down 75, always, for the first two figures, instead of 25; thus:

$175 \times 45 = 7875$:	Here, the sum of 17 and 4, or 21, is odd, and
195×65	75 is set down, instead of 25; now, 4 times 17 are
135×85	68, and 10 (half of 20) are 78. Or, add 8 (half 16)
etc.	and 2 (half of 4) as you proceed.

Now, the square of 695, or

$695 \times 695 = 483025$:
195×195
235×235
etc.

For squaring numbers ending in 5, the rule given on page 260 is preferable, since the units equal 10, and the other figures are alike always. Here, we say 5 times 5 are 25; set down in full; then adding 1 to 69, we say 70 times 69 are 4830 to complete the work.

The *reason* for adding half the sum will be understood if we take any two numbers ending in 5, say 85 and 45, and multiply their parts together, as shown in the margin.

$$\begin{array}{r} 80 + 5 \\ 40 + 5 \\ \hline 3200 + 400 \phantom{{}+25} \\ 200 + 25 \\ \hline 3200 + 600 + 25 \end{array}$$

Here, it will be seen that the partial products, when added, make 4825, or 85×45, and that, omitting the ciphers, we have 4 times 8, or 32, plus 6 (the half of $4 + 8$) plus 5 times 5. And a similar mode of reasoning will show why 75 must be set down, instead of 25, when the sum is odd.

The rule given on page 260, for multiplying together numbers of two or three figures whose units equal 10, the other figures being alike, can be applied to *mixed numbers whose fractions make* 1, *and whose whole numbers are alike;* thus:

$8\frac{1}{2} \times 8\frac{1}{2} = 72\frac{1}{4}$:
$7\frac{1}{8} \times 7\frac{7}{8}$
$6\frac{3}{4} \times 6\frac{1}{4}$
etc.

In this, the two fractions, when added, make 1, and the 8's are alike: say half by half is a quarter ($\frac{1}{4}$); then, add 1 to 8 and say 9 times 8 are 72. ($8\frac{1}{2} = 8.5$, and $8.5 \times 8.5 = 72.25$, or $72\frac{1}{4}$.)

RULE. (1) *Multiply the fractions together and set down the result.* (2) *Add* 1 *to either whole number and multiply the other by the number thus increased.*

$16\frac{3}{8} \times 16\frac{5}{8} = 272\frac{15}{64}$:
$14\frac{2}{3} \times 14\frac{1}{3}$
etc.

Here, the sum of $\frac{3}{8}$ and $\frac{5}{8}$ is 1, and the whole numbers are alike: now, $\frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$, which is set down; then, 16 times 17 (*short method*) are 272.

And any two mixed numbers, each having the fraction $\frac{1}{2}$, can be multiplied together by the following Rule: (1) *Multiply the fractions together and set down the result.* (2) *Multiply the whole numbers together and to their product add half their sum;* thus:

$9\frac{1}{2} \times 7\frac{1}{2} = 71\frac{1}{4}$:
$18\frac{1}{2} \times 6\frac{1}{2}$
etc.

In this, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, which is set down; then, 7 times 9 are 63 and 8 (half of $7 + 9$) are 71; the product is $71\frac{1}{4}$.

When the sum of the whole numbers is odd, add half the next lower even number, and set down $\frac{3}{4}$ for the fraction, always, instead of $\frac{1}{4}$; thus:

$17\frac{1}{2} \times 4\frac{1}{2} = 78\frac{3}{4}$: Here, 17 and 4 are 21, an odd number, set down $\frac{3}{4}$; now, 4 times 17 are 68, and 10 (half of 20) are 78.

$19\frac{1}{2} \times 6\frac{1}{2}$
etc.

This is the application of the rule given on page 276, for multiplying together numbers ending in 5.

GENERAL RULE. *To multiply ang two mixed numbers together: (1) Multiply the whole numbers together. (2) Multiply the whole number of the multiplicand by the fraction of the multiplier. (3) Multiply the whole number of the multiplier by the fraction of the multiplicand. (4) Multiply the fractions together; and add the four products.*

EXAM. Multiply $14\frac{1}{2}$ by $16\frac{3}{4}$.

In this, 16 times 14 (*short method*) are 224, which is set down; then, $\frac{3}{4}$ of $14 = 10\frac{1}{2}$; next, $\frac{1}{2}$ of $16 = 8$ and $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$: adding the four products, now, we get $242\frac{7}{8}$, the required product.

$$\begin{array}{r} 14\frac{1}{2} \times 16\frac{3}{4} \\ \hline 224 \\ 10\frac{1}{2} \\ 8 \\ \frac{3}{8} \\ \hline 242\frac{7}{8} \end{array}$$

The work can be done mentally, in most cases, without setting down the several products; thus: multiply $12\frac{3}{4}$ by $8\frac{1}{3}$.

Here, we say 8 times 12 are 96, and 4 ($\frac{1}{3}$ of 12) are 100; and 6 ($\frac{3}{4}$ of 8) are 106; then, $\frac{3}{4} \times \frac{1}{3} = \frac{3}{12}$, which gives...

$$\begin{array}{r} 12\frac{3}{4} \times 8\frac{1}{3} \\ \hline 106\frac{3}{12} \end{array}$$

Now, in actual business transactions, where the fraction of a cent cannot be taken (the rule being to reject the fraction when less than half a cent, and when a half, or more, to add a cent) the following simple method will answer all practical purposes: take $14\frac{1}{2}$ lbs. of tea, @ $16\frac{3}{4}$ ¢ per lb.

In this, we say $\frac{3}{4}$ of 14, to the nearest unit, is 11; and $\frac{1}{2}$ of 16 is 8; then, 16 times 14 are 224, and 19 (11 + 8) are 243, or \$2.43.

$$\begin{array}{r} 11 \quad\quad 8 \\ 14\frac{1}{2} \times 16\frac{3}{4} \\ \hline \$2.43 \end{array}$$

The correct answer is $2.42\frac{7}{8}$, as found above; but \$2.43 for business.

EXAM. What is the cost of $24\frac{1}{2}$ yds. of cloth, @ $23\frac{3}{4}$ ¢ per yard?

Here, $\frac{3}{4}$ of 24 to the nearest unit is 18; and the half of 23 to the nearest unit is 12; then 23 times 24 (*short method*) are 552, and 30 (18 + 12) are 582.

$$\begin{array}{c} 18 \qquad 12 \\ 24\frac{1}{2} \times 23\frac{3}{4} \\ \hline \$5.82 \end{array}$$

NOTE. — In multiplying by each fraction, always take what is nearest the true result; thus, the ½ of 23 is nearer 12 than 11, etc.

When either factor is an aliquot part of 100, 1000, etc., or is conveniently near an aliquot part, the multiplication can be more easily performed by division.

The aliquot parts of a number are obtained by dividing it by 2, 3, 4, 5, 6, etc., as shown in the margin, where the aliquot parts of 100 are given.

RULE. *To multiply by an aliquot part of* 100: *If there be no decimal in the multiplicand, annex two ciphers and divide by* 2, 3, 4, *etc., as the case may be. If there be decimals, or cents, move the decimal point two places to the right and divide as directed. If a mixed number, reduce the fraction to a decimal.*

	100		
2.	50	=	$\frac{1}{2}$
3.	$33\frac{1}{3}$	=	$\frac{1}{3}$
4.	25	=	$\frac{1}{4}$
5.	20	=	$\frac{1}{5}$
6.	$16\frac{2}{3}$	=	$\frac{1}{6}$
7.	$14\frac{2}{7}$	=	$\frac{1}{7}$
8.	$12\frac{1}{2}$	=	$\frac{1}{8}$
9.	$11\frac{1}{9}$	=	$\frac{1}{9}$
10.	10	=	$\frac{1}{10}$

EXAM. What is the cost of $16\frac{2}{3}$ yds. of cloth @ \$1.25 per yard?

Here, $16\frac{2}{3}$ being $\frac{1}{6}$ of 100, instead of multiplying in the usual way we move the decimal point two places to the right in \$1.25, this multiplies by 100, and gives \$125; now, $\frac{1}{6}$ of this gives \$20.83 the required cost.

$$\begin{array}{r} \$125 \\ \hline \$20.83 \end{array}$$

Or, since 125 is $\frac{1}{8}$ of 1000, by reducing the fraction in $16\frac{2}{3}$ to a decimal, we have 16.666′ to be multiplied by 125. Now, moving the point three places to the right, we have 16666. cents, and $\frac{1}{8}$ of this gives \$20.83 as before.

COMPUTING THE COST OF COMMODITIES.

In computing the cost of commodities, such as dry goods, etc., when the commodity contains a fractional part, the process can be simplified by reducing the fraction to a decimal and applying the method of aliquot parts.

Take for example $438\frac{3}{4}$ yards of goods at any particular price. By reducing $\frac{3}{4}$ to the decimal of a yard, we have $438\frac{3}{4}$ yds. = 438.75 yds.

Now, the cost of 438.75 yards at \$1 per yard =	\$438.75
and at 10 cents per yard, the cost is one-tenth of that =	43.875
at 1 cent per yard, the cost is one-tenth of that at 10 cents =	4.3875
and at $\frac{1}{2}$ cent, the cost is half that at 1 cent =	2.1937

And from this basis the cost at any given price can be easily obtained, illustrated in the following examples:

EXAM. 1. Find the cost of $438\frac{3}{4}$ yds. of silk at \$$1.37\frac{1}{2}$ per yd.

In this $438\frac{3}{4}$ yds. = 438.75, and at \$1 per yd. the cost is	\$438.75
Now, $37\frac{1}{2}$c. = 25c., plus $12\frac{1}{2}$c.; and 25c. = $\frac{1}{4}$ of \$1 = ...	109.69
and at $12\frac{1}{2}$c. the cost is one-half that at 25c. = ...	54.84
making the total cost \$603.28.	\$603.28

EXAM. 2. What is the cost of $86\frac{7}{8}$ yds. at \$$1.38\frac{1}{2}$ per yd.

Here we have $86\frac{7}{8}$ yds. = 86.875 yds. and proceeding as	\$86.875
in the foregoing example, we obtain the three first items	21.718
which give the cost at \$$1.37\frac{1}{2}$; and the cost at 1c. is	10.859
found from the first item (\$86.875) by moving the point	.868
two places to the left; this gives .868 making the cost \$120.32.	\$120.32

EXAM. 3. What is the cost of $67\frac{5}{8}$ yds. at \$$1.87\frac{1}{2}$ per yd.?

In this, $67\frac{5}{8}$ yds. = 67.625; and at \$1, the cost = ...	\$67.625
Now, the difference between \$$1.87\frac{1}{2}$ and \$2, is $12\frac{1}{2}$c., so	135.25
we multiply by 2 to get the cost at \$2 per yd. and	8.453
from this we deduct one-eighth of the cost at \$1, or \$8.453, the cost at $12\frac{1}{2}$c. ($\frac{1}{8}$ of \$67.625 = \$8.453) to get the cost at \$$1.87\frac{1}{2}$.	\$126.797

NOTE.—To reduce the fractional part to a decimal, annex ciphers to the numerator, or suppose them annexed, and divide by the denominator.

ADDITION.

The following method for Addition requires no "carrying," and will be of advantage to accountants when liable to be interrupted in their calculations:

```
  3829.25
   768.50
  4687.49
  2823.35
  7547.28
  3760.82
   675.64
  1846.35
  3785.10
 --------
 23270.48
  6453.3
 --------
 29723.78
```

Commencing at the top of the *left-hand* column and running downwards, we find the sum to be 23, which is set down in full, in the usual manner; then, *without carrying*, we find the sum of the next column to be 62; 2 is set in its proper place under the column, and 6 to the left, one line lower; the sum of the next column, without carrying, is 47; the next 50; then 34, and finally 38. The sum of the two results thus obtained is the required sum.

NOTE.—It is preferable to write the numbers of each sum in the natural order, thus: 62, write down 6 first and then 2, etc.

Instead of taking one column into consideration, as in the foregoing, let us take two columns, as in the following example:

```
 7638.42
 3893.87
 6324.73
  139.80
 7627.25
 2563.87
 7326.24
   67.38
  735.92
  682.43
 2756.87
--------
39250.78
  5 6
--------
39756.78
```

Commencing at the bottom of the left-hand column and running up, we find the sum to be 34; call this 340; now run down the next column, starting with 40, bearing in mind the 3 (300), by taking hold of the third finger of the left hand; the sum of the two columns is 392, which is set down in full. Next, take the third column from the left, without carrying, the sum is 49, call this 490 and run down the fourth column starting with 90; the sum of both columns is 550, which is set down in full, 50 in proper position, and 5, that is 500, to the left and one line lower; the sum of the next two columns is 678, which is set down as shown, and the total sum is 39756.78.

NOTE.—If the number of columns be odd, first add the left hand column, then two columns at a time, thus:

The left-hand column is 26, the sum of the two next, 298; and the sum of the two last, 324.

```
 764.54
 123.45
 678.93
 876.43
 457.89
-------
2698 24
 2 3
-------
2901.24
```

The following rule for Subtraction will be found simple and practical : RULE. — *Add to the minuend, first, what the unit figure of the subtrahend wants of being* 10; *and for the succeeding figures, add what each wants of being* 9. *Drop* 1 *immediately to the left of the last figure of the subtrahend always.*

EXAM. 1. From 74721 take 39864.

Here, we say 6 and 1 are 7; 3 and 2 are 5; 1 and 7 are 8; 4 is 4; 6 and 7 are 13; drop the 1, and the remainder is 34857.	74721 39864 (1)34857

EXAM. 2. From the sum of $687463 and $2346 ; take $6942.

$687463 + $2346 − $6942 = $682867.

In this, we say 8 and 6 are 14, and 3 are 17; 7 and 1 to carry: 1 and 5 are 6, and 4 are 10, and 6 are 16; 6 and 1 to carry; 1 and 3 are 4 and 4 are 8; 3 and 2 are 5, and 7 are 12; 2 and 1 to carry; 1 and 8 are 9 ; but dropping 1 immediately to the left of the subtractive number, we set down 8; then setting down the last figure, 6, the remainder is $682867.

EXAM. 3. Received $756.47 ; $982.34 ; $765.26 ; and paid out $476.29 ; what is the balance on hand?

In this example, we set the subtractive number under those to be added, and perform the whole process by addition, adding mentally, what the first figure of $476 29 wants of being 10; and what each remaining figure wants of being 9. The sum of the last column is 30; but dropping 1 from 3, immediately to the left of $476.29, makes it 2.	$756.47 982.34 765.26 476.29 $2027.78

The *reason* of dropping 1 to the left of the last figure of the subtrahend, always, will be understood from the following :

Any number, which, when added to another, makes 10, 100, 1000, 10000, etc., is called the complement of that other. Thus, 43+57=100; 43 is the complement of 57; and 57 is the complement of 43. The complement of $476.29 is $523.71, both numbers, when added, making $1000.

Subtracting either number, now, from 1000, is the same as adding its complement and dropping the 1000.

Thus, subtracting $476.29 from $1000 leaves $523.71; the same as adding $523.71 and subtracting $1000, or dropping the 1 in 1000.

$1000	$1000
476.29	523.71
$523.71	(1)523.71

And this being understood, any number can be subtracted from another, by adding (mentally) the complement of the number to be subtracted, dropping 1 to the left of that number, always. (*See page* 221.)

EXAM. 1. From 547632 take 876.

In this, we say, 4 and 2 are 6; 2 and 3 are 5; 1 and 6 are 7; here we drop 1, and set down 6; then 4, and 5.

547632
876
546756

EXAM. 2. From 370234 take 18547.

Here, we say, 3 and 4 are 7; 5 and 3 are 8; 4 and 2 are 6; 1; 8 and 7 are 15; carry 1 to 3 is 4, but here 1 is to be dropped, and we set down 3.

370234
18547
351687

The following problems, with their solutions, are given for the purpose of showing the student how they and similar problems may be solved.

PAPER PROBLEMS.

PROB. 1. What is the cost of 187 sheets 25 lb. paper, 500 sheets to the ream, @ 18c per lb.?

For the solution of this and similar problems, we give the following simple

RULE.— *Multiply the number of sheets by twice the weight of the ream, and the result by the price; or by twice the price, and the result by the weight, whichever is most convenient, and point off five decimal places, always.*

Solution. — $187 \times 50 \times 18 = \$1.68300$; or, $187 \times 25 \times 36 = \$1.68300$; for business $1.68.

The *reason* for pointing off five places: If we proceed in the usual way we multiply the number of sheets by the weight and the result by the price, and divide by 500.

Now, by doubling the weight, or the price, we double 500, also, making 1000 for the divisor. Three places are cut off for the ciphers in 1000, and two for .18 in the multiplication, making five places in all.

NOTE. — If the ream consist of 480 sheets, proceed according to the rule, and add to the result four per cent. of itself (.04) thus, $\$1.68300 \times .04 = .06732$; then,

$$\begin{array}{r} \$1\ 683 \\ .067 \\ \hline \$1.75 \end{array}$$

Reason: The difference between 500 and 480 = 20, and when we divide by 500 instead of 480, we obtain a result too small by $\frac{20}{500}$ or $\frac{4}{100} = .04$.

PROB. 2. 28 lb. paper is listed to be sold at 10c. per lb. by the ream, but for a quantity purchased less than a ream, 25% extra is charged; what is the cost of 264 sheets on these terms?

Solution. — $264 \times 28 \times 20 = \$1.47840$; then $25\% = \frac{1}{4}$ and we have $1.47840 plus

$$\begin{array}{rr} \frac{1}{4} = & .36960 \\ \hline & \$1.84800 \end{array}$$

Or,

By adding the percentage to the number of sheets, the weight, or the price, whichever is most convenient, we obtain the same result, thus, 264 plus $\frac{1}{4}$ or $66 = 330$, and we have $330 \times 28 \times 20 = \$1.84800$; or, by adding 25% to 28 making it 35, we have $264 \times 35 \times 20 = \$1.84800$.

PAPER PROBLEMS.

Weights of paper equal in thickness to 24 x 36.

RULE. — *To find the weights of paper equal in thickness to* 24×36*:* (1.) *Draw a vertical line, and on the left of said line, set the given dimensions; and on the right set the required dimensions, and the given weight.* (2.) *Divide the product of the numbers on the right by the product of those on the left; the result is the required weight.*

EXAM. If 24 x 36 weigh 70 pounds; what will 40 x 48 weigh?

Solution. —
$$\begin{array}{r|l} 24 & 40 \\ 36 & 48 \\ & 70 \\ \hline 864 & 134400(155.5 \end{array}$$

It is scarcely necessary to remark that recourse may be had to cancellation in the majority of such cases, thereby shortening the process; thus:

$$\begin{array}{r|l} 1 & 10 \\ \cancel{24} & \cancel{40} \\ \cancel{36} & \cancel{48}\,2 \\ & 70 \\ \hline 9 & 1400 \\ \hline & 155.5 \end{array}$$

Dividing 24 and 48 each, by 24, we obtain 1 and 2; and dividing 36 and 40 each, by 4, we obtain 9 and 10. We have now $10 \times 2 \times 70 \div 9 = 155.5$.

PERCENTAGE PROBLEM.

If $30,000 be invested in property which rents for $250 per month, and on which $750 are paid in taxes; what rate of interest does the investment pay?

Solution. — Here, $250 per mo. is $3,000 per year, out of which $750 are paid in taxes leaving $2,250 income on the investment.

Now, if $30,000 in one year give $2,250 income, what income (rate) ought $100 give?

And for this we have the following proportion:

As $30,000 : $100 : : $2,250 : x, or the rate on $100; and we have $225,000 ÷ $30,000 = $7\frac{1}{2}$%. (*See Useful Rules, pages* 155 *to* 158.)

RULE. — *Divide* 100 *times the income by the investment.*

THE POPULATION PROBLEM.

If the population of Albany was 90,000 in the year 1900, and 100,000 in 1910; what was the rate per cent increase during the interval?

Solution. — Here 100,000 — 90,000 = 10,000, the increase of population during the interval.

And we have: As 90,000 : 100 : : 10,000 : x = 1,000,000 ÷ 90,000 = $11\frac{1}{9}$%. (*See page* 158)

RULE. — *Divide* 100 *times the increase of population during the interval, by the population of the earlier date; the result is the rate per cent.*

THE GRANT PROBLEM.

General Grant sold 2 horses for \$198 each, he gained 10% on one and lost 10% on the other, what was the result of the sale?

Solution.— In this, rule 111 of Profit and Loss, page 144, is applicable: To find the first cost from the gain per cent., and the selling price.

Now, what cost \$100 was sold for \$110 and we have to find what the horse *cost* which was sold for \$198.

As the selling price, \$110 : \$100 cost, : : \$198 selling price to x, or the cost, and we have $100 \times 198 \div 110 =$ \$180, the first cost.

Selling price \$198, cost price \$180; the result was a gain of \$18 in the first place.

Next, what cost \$100 was sold for \$90; and we have the following As \$90 : \$100 : : \$198 : x = 19800 ÷ 90 = \$220, the cost in the second place, showing a loss of \$22; then \$22 — \$18 = \$4 loss, the result of the sale. (*See problem* 2, *page* 145.)

THE SHIP PROBLEM.

A ship springs a leak 40 miles from shore, and admits $3\frac{3}{4}$ tons of water in 12 minutes; 60 tons will sink her, but the pumps throw out 12 tons per hour. Find the average rate of speed to bring her to shore before sinking.

Solution. — The ship admits $3\frac{3}{4}$ tons in 12 min., she will admit 5 times that in 60 min., or $18\frac{3}{4}$ tons per hour ($3\frac{3}{4} \times 5 = 18\frac{3}{4}$ tons) but the pumps throw out 12 tons per hour: $18\frac{3}{4} - 12 = 6\frac{3}{4}$ tons, the quantity admitted per hour. Now, if $6\frac{3}{4}$ tons be admitted in 1 hour, how long will it take to admit 60 tons?

The proportion is as follows:

As $6\frac{3}{4} : 60 : : 1 : x = 60 \div 6\frac{3}{4} = 8\frac{8}{9}$ hours.

Next, if in $8\frac{8}{9}$ hours the ship has to go 40 miles, what must be the speed per hour?

As $8\frac{8}{9}$ h. : 1 h. : : 40 m. : $x = 40 \div 8\frac{8}{9} = 4\frac{1}{2}$ miles.

Proof: $8\frac{8}{9} \times 4\frac{1}{2} = 40$ miles.

THE OIL PROBLEM.

A quantity of flax seed being converted into oil, the result was found to be 658 lbs. of oil and 1276 lbs. of cake; how much oil is that to the bushel; what per cent.; and what is the value of the oil at 10¢ per gallon; a bushel of seed being 56 lbs., and a gallon of oil $7\frac{1}{2}$ lbs.?

Solution. — Here, the rule given on page 149 is applicable, thus, $658 + 1276 = 1934$, the whole number of pounds obtained from the seed.

Now, as 1934 : 658 : : 56 : x; and we have $658 \times 56 \div 1936 = 19.05$ lbs. to the bushel.

Dividing this by $7\frac{1}{2}$ lbs. to the gal. we have $19.05 \div 7\frac{1}{2} = 2.54$ gal. or $2\frac{1}{2}$ gal. nearly to the bushel.

Next, we have 1934 : 658 : : 100 : x, or the percentage, that is, we have $65800 \div 1934 = 34\%$ nearly.

Finally, dividing the number of pounds of oil, viz., 658 by $7\frac{1}{2}$ gives the number of gallons; $658 \div 7\frac{1}{2} = 87.7$ gals. nearly, and at 10¢ per gal. the value is $8.77.

NOTE. — To divide by 7½, add one-third and take one-tenth. (*See exam.* 1, *p.* 249.)

THE CHICKEN PROBLEM.

Do figures lie? Let us see.

Two women sold 30 chickens each and agreed to divide the proceeds equally. One sold her chickens 2 for $1, getting $15 for 30 chickens and the other sold hers 3 for $1, getting $10 for her 30 chickens. This made $25 for 60 chickens.

The merchant being asked to divide the money, said: You sold yours 2 for $1; and yours, 3 for $1, that is 5 for $2; well 5 into 60, 12 times and 12 times 2 are $24; $12 each.

But the women received $25; how can this be explained?

Solution. — 2 for $1 = 1 for 50¢
3 for $1 = 1 for $33\frac{1}{3}$¢
2 for $83\frac{1}{3}$¢ or 1 for $41\frac{2}{3}$¢ average price.

Now, 60 @ $41\frac{2}{3}$¢ = $25.

THE CONTRACT PROBLEM.

Four men contracted to do a certain job of work for \$8600; the first employed 28 men 20da., 10 h. a day; the second, 25 men 15 da., 12 h. a day; the third, 18 men 25 da., 11 h. a day; and the fourth, 15 men 24 da., 8 h. a day. How much should each contractor receive?

Solution. — $28 \times 20 \times 10 = 5600$, the number of hrs. for 1st contractor.
$25 \times 15 \times 12 = 4500$, " " " 2nd "
$18 \times 25 \times 11 = 4950$, " " " 3rd "
$15 \times 24 \times 8 = 2880$, " " " 4th "
17930, total number of hours for \$8600.

And here the rule given on page 149, for division into proportional parts is applicable; the question resolving itself into this:

If \$8600 be paid for 17930 hours' of work; what should be paid for 5600 h., 4500 h., 4950 h., and 2880 h.? For this we have the following:

1st. As $17930 : 5600 : : 8600 : x = \frac{5600 \times 8600}{17930} = \$2686.00$

2nd. As $17930 : 4500 : : 8600 : x = \frac{4500 \times 8600}{17930} = 2158\ 39$

3rd. As $17930 : 4950 : : 8600 : x = \frac{4950 \times 8600}{17930} = 2374.24$

4th. As $17930 : 288 : : 8600 : x = \frac{2880 \times 8600}{17930} = 1381.37$

\$8600.00

THE COAT PROBLEM.

A boy agreed to work for a mechanic 20 weeks, on condition that he should receive \$20 and a coat. At the end of 12 weeks the boy quit work and received \$9 and the coat. What was the value of the coat?

Solution. — The boy's loss in 8 weeks (20−12) was \$11 (20−9). If he had worked the 8 weeks he should have received \$11 and \$9.

Now, if for 8 weeks he should have received \$11; what should he receive for 12 weeks?

$$\text{As } 8 : 12 : : 11 : x = \frac{12 \times 11}{8} = \$16.50$$

he should have received \$16.50; but he received only \$9, the coat, therefore, was worth the difference \$7.50.

\$16.50
9.00
\$7.50

MARKING GOODS.

Short methods for *trade discounts* have already been given from page 232 to 237. In connection therewith, it may be well to give here a few examples on the marking of goods, calling attention to the fact that *losses* may be sustained when supposed *profits* are being made.

EXAM. 1. Suppose goods are marked to sell at 40% above cost, and we offer a discount of 20%, our profit is not 20% but 12%; for the reason the discount is *not* calculated on the *cost* of the goods but upon the *marked or asking price*, which includes the first cost and the per cent. of profit.

Solution.— What cost $100, we have marked at $140
and on this we allow a discount of 20%, or $\frac{1}{5}$...... 28
showing a profit of only 12%. $112

Again, suppose, without full consideration, we offered a discount of 30% on the foregoing, instead of 20%; what would be the result?

Here, what cost $100, we have marked to sell for............$140
and 30% of this is found to be $42, which on being deducted ... 42
shows a loss of $2. $98

RULE.—*To mark goods so as to allow a discount on the asking price and have them net a certain profit:*

Divide the net, or desired price by 100 less the discount.

EXAM. 2. What must be the asking price of goods, to allow the purchaser a discount of 20% and net the manufacturer $25 per dozen?

Solution.—Here, we have 100%−20%=80%; decimally expressed, .80. Dividing the desired price, $25, by .80, we have 2500 ÷ 80 = $31.25. Or,

Applying the rule given under the head of percentage Case V, page 118; the question may be asked thus: What number diminished by 20% of itself is equal to 25? And for this we have the following proportion:

As 80 : 100 : : 25 : x = 2500 ÷ 80 = $31.25
Proof: 20% = $\frac{1}{5}$ and $\frac{1}{5}$ of $31.25 = 6.25
deducting $6.25 we obtain $25.00

EXAM. 3. What must be the asking price of a piano costing $350 to allow a discount of 25% to the buyer and still gain 20% above cost?

Solution.—To realize the desired profit in this, 20%, or $\frac{1}{5}$ of the cost, is added to itself; this gives $420, or the net price at which the manufacturer wishes to sell the instrument.

	$350.00
	70.00
	$420.00

Now, what sum diminished by 25% of itself is equal to $420?

As $75 : $100 : : $420 : x = $42000 ÷ 75 = $560.00 the asking price.

Hence the following:

RULE.—*Add the desired profit, or rate per cent. to the cost price, multiply this by 100* (annexing two ciphers) *and divide the result by 100 less the rate of discount.*

Proof: Marked price	$560
Less 25% discount	140
Net selling price	$420
Less added profit	70
Cost	$350

RULE.—*To find what rate per cent. of discount may be allowed on the asking price to realize cost: Subtract the cost from the asking price, annex two ciphers, and divide the result by the asking price.*

EXAM. 4. If the cost of a piano be $350 and the asking price $560; what rate per cent. of discount can be offered to realize cost?

Solution.— $560 — $350 = $210; then $21000 ÷ 560 = $37\frac{1}{2}$%.

Case 2 of percentage, page 115, is applicable here: Given the percentage and base to find the rate.

Deducting the cost from the asking price gives the percentage. In the present example, 560 is the base and 210 the percentage; and the problem may be resolved into this: If the percentage on 560 be 210; what is the percentage on 100? And for this we have the following proportion:

As 560 : 210 : : 100 : x = 21000 ÷ 560 = $37\frac{1}{2}$%.

This explains the foregoing rule.

Proof: $37\frac{1}{2}$% of 560 = $210 and	$560
Less	210
Cost	$350

RULE.— *To tell quickly what a single article should sell for, when commodities are bought by the dozen, to gain a certain per cent. of profit: Divide the cost per dozen by 10; the result will be the selling price including a profit of 20% always; and from this any per cent. of profit may be readily obtained; illustrated in the following examples:*

EXAM. 1. If hats be bought for \$35 a dozen; what must be the selling price of a single hat to make a profit of 20%? Ans. $\frac{1}{10}$ of \$35 = \$3.50.

The *reason* for dividing by 10 to obtain a profit of 20%, will be understood from the following proportion:

As \$100 considered as cost, is to \$120, selling price, so is \$35 cost to its corresponding selling price; and as 12 articles is to 1, so is the cost of 12 to the selling price of 1; thus:

Reducing this to a simple proportion, and cancelling 1200 and 120, we obtain 10 to 1 as 35 to x, and dividing 35 by 10, we get the selling price of a single hat including a profit of 20%. (*See exam.* 3, *page* 146; *and note page* 147.)

$$\begin{array}{l} \text{As } 100 : 120 : : 35 : x \\ \underline{\quad 12 \quad} : \underline{\ \ 1 \ \ } \\ 1200 : 120 : : 35 : x \\ \quad 10 : \quad 1 : : 35 = 35 \div 10 = \$3.50 \end{array}$$

EXAM. 2. If shoes be bought for \$32 a dozen pairs; what should be the selling price of one pair to realize a profit of 50%?

Solution.—To make a profit of 20%, we take one-tenth of \$32=\$3.20 and this represents 120%; but we desire a profit of 50%, or a selling price of 150%; the difference is 30% (150—120) and 30% is one-fourth of 120%, so we add to \$3.20 one-fourth of itself, or 80¢; this gives the selling price of a pair, \$4.

$$\begin{array}{r} \$3.20 \\ .80 \\ \hline \$4.00 \end{array}$$

Proof: 12 pairs cost	\$32.00
Added profit, 50%....	16.00
Selling price of 1 pair	\$48.00 ÷ 12 = \$4

NOTE.— To make 60% profit, add $\frac{1}{3}$ of itself to the 20% (160—120=40) and 40% is $\frac{1}{3}$ of 120%; to make 30%, add $\frac{1}{12}$ (130 — 120 = 10) and 10% is $\frac{1}{12}$ of 120%; for $33\frac{1}{3}$%, add $\frac{1}{9}$ ($133\frac{1}{3}$ — 120 = $13\frac{1}{3}$) and $13\frac{1}{3}$% is $\frac{1}{9}$ of 120%, etc. And when the desired profit is less than 20%, deduct, thus: To make $16\frac{2}{3}$%, deduct $\frac{1}{36}$ (120 — $116\frac{2}{3}$=$3\frac{1}{3}$) and $3\frac{1}{3}$ is $\frac{1}{36}$ of 120%; to make 15%, deduct $\frac{1}{24}$ (120—115=5) and 5% is $\frac{1}{24}$ of 120%, etc.

INVOLUTION.

Involution is the process of raising a number to any proposed power.

A Power of a number is either the number itself, or the product arising from using the number a certain number of times as a factor.

The *first power* of a number is the number itself. Thus, the first power of 5 is 5. When the number is used *twice* as factor, the result is the *second power*, or the *square*, of that number; when *three times*, the *third power* or cube; when *four times*, the *fourth power*, etc.

Powers are denoted by a small figure placed above and to the right of the number, to show how many times it is taken as a factor; thus, $5^1 =$ the first power of 5.

$5^2 = 5 \times 5 = 25$, the second power, or square of 5.

$5^3 = 5 \times 5 \times 5 = 125$, the third power, or cube of 5.

$5^4 = 5 \times 5 \times 5 \times 5 = 625$, the fourth power of 5.

The small figure above and to the right of the number is called the *Index*, or the *Exponent*, of the power.

NOTE.—The number of multiplications is one less than the numbar of factors As the first power of a number is the number itself, its index or exponent is generally understood.

PRINCIPLES I. *A number is raised to a given power by taking it as a factor as many times as there are units in the exponent; thus,* $5^3 = 5 \times 5 \times 5 = 125$.

II. *The product of any two or more powers of the same number, is equal to the power indicated by the sum of their exponents thus,* $5^2 \times 5^3 = (5 \times 5) \times (5 \times 5 \times 5) = 5^{2+3} = 5^5$.

NOTE.—It frequently happens in the multiplication of decimals, or in raising a decimal to a proposed power, that a greater number of decimal figures is obtained in the product, than is necessary for practical accuracy. This may be avoided by making use of the following *contracted method* for the multiplication of decimals:

RULE. (1) *Count off, after the decimal point in the multiplicand, (annexing ciphers, if requisite) as many figures of decimals as it is necessary to have in the product.* (2) *Below the last of these, write the unite figure of the multiplier, and write the other figures in reversed order.* (3) *Then multiply by each figure of the multiplier, thus inverted, neglecting all the figures of the multiplicand to the right of that figure, except to find what is to be carried; and let all the partial products be so arranged, that their right hand figures may stand in the same column.* (4) *Lastely, from the sum of the partial products, cut off the assigned number of decimal places.*

EXAM. Multiply 7.24651 by 81.4632, so that there may be three decimal places in the product.

CONTRACTED METHOD.	COMMON METHOD.
7.24651	7.24651
2364.18	81.4632
579721	1|449302
7247	21|73952
2898	434|7906
435	2898|604
22	7246|51
1	579720|8
590.324	590.323|893432

Here 1, the unit figure of the multiplier, is set below 6, the third decimal figure of the multiplicand; 8, the figure which *precedes* 1, is put *after* it; 4, the figure which *follows* it, is set before it, etc. We then say, 8 times 5 are 40, and 1 (carried from 8 times 1) are 41: 1 is then set down and 4 carried, and the rest of the work by 8 proceeds in the usual way. Then in multiplying 7.246 by 1, we add 1 to the product for 51 because 51 is nearer 100 than 0, and therefore it is nearer the truth to carry 1 than 0. In multiplying 7.24 by 4, 3 is carried for the product that would have resulted from the rejected figures: for, going two places back, we have 4 times 5 are 20; 4 times 6 are 24, and 2 are 26, which being nearer 30 than 20, we carry 3.

So likewise in multiplying 7.2 by 6, we carry 3 from the rejected figures, and thus we proceed in similar cases. In finding what is to be carried from the rejected figures, it is generally sufficient to go one place back, but in doubtful cases it may be well to go farther.

The *reason* of the contracted method will be seen from the common method, in which a vertical line is drawn, cutting off the part rejected in the contracted method.

To illustrate the contracted method still further, let us take the following

EXAM. Multiply .681472 by .01286, so that the decimal in the product may contain five figures.

CONTRACTED METHOD.	COMMON METHOD.
.6814 72	.681472
68210.0	.01286
681	4|088832
136	54|51776
55	136|2944
4	681|472
.00876	.00876|372992

In this example, since the multiplier contains no integer, a cipher is placed below the fifth figure of the multiplicand; and then, the multiplier being written in reversed order, the work proceeds as in the last example. A glance at the common method will show the advantage of the contracted method.

The contracted method will be particularly useful in computations in Compound Interest, given on page 296, and illustrated in the following.

EXAM. What will \$1 amount to in 10 years at 6% per annum, compound interest?

Solution.— The amount of \$1 for one year at 6% is \$1.06, and by raising this to the tenth power, we get the amount for 10 years.

$1.06 \times 1.06 = 1.1236$ = the second power; and this multiplied by itself will give the fourth power (*Prin. II*); next, the fourth multiplied by itself will give the eighth power; finally, the eighth multiplied by the second, will give the tenth power.

NOTE. — In carrying from the rejected figures, we should take what is *nearest* the truth whether it be too great or too small.

In computations in compound interest, the decimal figures are usually carried to six places; so we annex two ciphers to 1.1236 so that the product may contain six. Then reversing 1.1236, setting the unit figure under the sixth figure of the decimal and multiplying by the contracted method, we obtain the fourth power. Next, reversing these figures, and multiplying we obtain the eighth power; and finally, multiplying this by the second power reversed, we obtain the tenth power, which is the amount of \$1 for 10 years at 6%. And from this the amount of any sum may be obtained by multiplication.

1.12360 0
6321.1
1 12360 0
11236 0
2247 2
337 1
67 4
1.26247 7 = 4th
7 74262 1
1 26247 7
25249 5
7574 9
252 5
50 5
8 8
9
1.59384 8 = 8th
6321.1 = 2d
1 59384 8·
15938 5
3187 7
478 2
95 6
1.79084 8 = 10th

Had the products here been found at full length the work would have been quite laborious, requiring 346 figures in the operation, and giving 21 figures for the last product, 1.79084769654285362176; whereas by the contracted method we obtain 1.790848, six decimal figures sufficient for all practical purposes.

To facilitate the calculation of such problems as the foregoing, however, we give on page 298 a table showing at sight the amount of \$1 at 2% to 10% for any number of years from 1 to 35.

Thus, taking the above example, to find the amount of \$1 for 10 yrs. @ 6 per cent. per annum, compound interest; we turn to the table and look in the yearly column for 10 years, and opposite in the 6 % column, we find 1.790848.

COMPOUND INTEREST.

Compound Interest is interest on both principal and interest, when the interest is not paid when due.

RULE.— *When the interest is payable annually:*

I. *Find the amount of the given principal for one year at the given rate, and make it the principal for the second year.*

II. *Find the amount of this new principal, and make it the principal for the third year, and so continue for the given number of years.*

III. *Subtract the given principal from the last amount; the remainder will be the compound interest.*

NOTES.—1. When the interest is payable semi-annually or quarterly, find the amount of the given principal for the first interval, and make it the principal for the second interval, proceeding in all respects as when the interest is payable annually.

2. When the time contains years, months, and days, find the amount for the years, upon which compute the interest for the months and days, and add it to the last amount before subtracting.

EXAM. What is the compound interest of \$800 for 4 years at 5%?

The amount of \$1 for 1 year at 5% = \$1.05, and this multiplied by the given principal will be the amount of said principal for 1 year. The solution is as follows:

	\$800	Principal for 1st year.
\$800 × 1.05 =	\$840	" " 2d "
\$840 × 1.05 =	\$882	" " 3d "
\$882 × 1.05 =	\$926.10	" " 4th "
\$926.10 × 1.05 =	\$972.40	Amount for 4 years.
	\$800	Given principal.
	\$172.40	Compound interest.

If the amount of \$1 for 1 year be raised to the power denoted by the number of years, the result will be the amount of \$1, or £1, for the number of years.

Thus, $1.05 \times 1.05 \times 1.05 \times 1.05 = 1.215506$, and this multiplied by the given principal gives the amount: $1.215506 \times 800 = \$972.40$, as found above.

As calculations in compound interest are facilitated by the use of interest tables, we give on pp. 298 and 299 a table showing the amount of \$1, or £1 sterling, at compound interest for any number of years from 1 to 35. Thus, taking the foregoing problem: Looking in the yearly column we find 4 yrs. and opposite in the column for 5% we find 1.215506.

EXAM. What is the amount of \$6000 for 10 years at 5% compound interest, payable semi-annually?

Here, the payments being half-yearly, the rate is $2\frac{1}{2}$ or .025 and the amount of \$1 for half a year is 1.025; and since there are 20 payments, 1.025 is to be raised to the 20th power. To avoid such a long operation, we refer to the table to 20 yrs. and $2\frac{1}{2}\%$ and we find 1.638616, the amount of \$1, and multiplying this by the prin. we have the required amount, thus, $1.638616 \times 6000 = \$9831.696$.

Since the *time* used in expressing *any rate* of interest is entirely arbitrary, and having fixed the ratio between the principal and interest at each compounding, the result evidently depends upon the *number* of times the operation is repeated.

Thus, if the interest be compounded a given number of times by adding to each respective amount 5% of itself, it matters not whether it be considered 5% per annum or 5% per day, the result would be the same.

Hence, if the intervals be less than a year, as when the interest is to be compounded semi-annually or quarterly, the tables constructed for yearly intervals can be used by reducing the rate per cent. proportionally, and taking from the table the proper number of intervals. If the interest is to be compounded semi-annually, when the rate is said to be 5% per annum, $2\frac{1}{2}\%$ should be used at each compounding, though it would amount to more than 5% compounded annually.

EXAM. Required the amount of \$5000 for 10 yrs., 8 mo., 20 da. @ 6% per annum, compound interest, payable semi-annually.

Solution.—In 10 yrs., 6 mo. there are 21 intervals of six months each, leaving 2 mo. and 20 da.; and the rate is 3% semi-annually.

The amount of \$1 at 3% half yearly is 1.03, and this raised to the 21st power (1.03^{21}) gives the amount of \$1 for 21 intervals, or for 10 yrs., 6 mo. compound interest. Multiplying this amount by the given principal, \$5000, gives the amount of said principal for 10 yrs., 6 mo. and adding to this amount its interest for 2 mo., 20 da. @ 6%, we get the total amount.

Now, to save time and labor, we refer to the table given on page 298, and look in the yearly column for 21 years, and opposite, in the 3% column, we find 1.860295, the amount of \$1, and multiplying this by the given principal, we have: $1.860295 \times 5000 = \$9301.48$, the compound amount for 10 yrs., 6 mo. Adding to this its interest for 2 mo., 20 da. @ $6\% = 124.02$, we obtain \$9425.50, the required amount.

Showing the amount of $1, or £1, compound interest, from 1 to 35 years, at the following rates:

Years.	2%.	2½%.	3%.	3½%.	4%.	4½%
1	1.020000	1.025000	1.030000	1.035000	1.040000	1.045000
2	1.040400	1.050625	1.060900	1.071225	1.081600	1.092025
3	1.061208	1.076891	1.092727	1.108718	1.124864	1.141166
4	1.082432	1.103813	1.125509	1.147523	1.169859	1.192519
5	1.104081	1.131408	1.159274	1.187686	1.216653	1.246182
6	1.126162	1.159693	1.194052	1.229255	1.265319	1.302260
7	1.148686	1.188686	1.229874	1.272279	1.315932	1.360862
8	1.171659	1.218403	1.266770	1.316809	1.368569	1.422101
9	1.195093	1.248863	1.304773	1.362897	1.423312	1.486095
10	1.218994	1.280085	1.343916	1.410599	1.480244	1.552969
11	1.243374	1.312087	1.384234	1.459970	1.539454	1.622853
12	1.268242	1.344889	1.425761	1.511069	1.601032	1.695881
13	1.293607	1.378511	1.468534	1.563956	1.665074	1.772196
14	1.319479	1.412974	1.512590	1.618695	1.731676	1.851945
15	1.345868	1.448298	1.557967	1.675349	1.800944	1.935282
16	1.372786	1.484506	1.604706	1.733986	1.872981	2.022370
17	1.400241	1.521618	1.652848	1.794676	1.947901	2.113377
18	1.428246	1.559659	1.702433	1.857489	2.025817	2.208479
19	1.456811	1.598650	1.753506	1.922501	2.106849	2.307860
20	1.485947	1.638616	1.806111	1.989789	2.191123	2.411714
21	1.515666	1.679582	1.860295	2.059431	2.278768	2.520241
22	1.545980	1.721571	1.916103	2.131512	2.369919	2.633652
23	1.576899	1.764611	1.973587	2.206114	2.464716	2.752166
24	1.608437	1.808726	2.032794	2.283328	2.563304	2.876014
25	1.640606	1.853944	2.093778	2.363245	2.665836	3.005434
26	1.673418	1.900293	2.156591	2.445959	2.772470	3.140679
27	1.706886	1.947800	2.221289	2.531567	2.883369	3.282010
28	1.741024	1.996495	2.287928	2.620172	2.998703	3.429699
29	1.775845	2.046407	2.356566	2.711878	3.118651	3.584036
30	1.811362	2.097568	2.427262	2.806794	3.243398	3.745318
31	1.847589	2.150007	2.500080	2.905031	3.373133	3.913857
32	1.884541	2.203757	2.575083	3.006708	3.508059	4.089981
33	1.922231	2.258851	2.652335	3.111942	3.648381	4.274030
34	1.960676	2.315322	2.731905	3.220860	3.794316	4.466362
35	1.999890	2.373205	2.813862	3.333590	3.946089	4.667348

Showing the amount of $1, or £1, compound interest, from 1 to 35 years, at the following rates:

Years.	5%.	6%.	7%.	8%.	9%.	10%
1	1.050000	1.060000	1.070000	1.080000	1.090000	1.100000
2	1.025000	1.123600	1.144900	1.166400	1.188100	1.210000
3	1.157625	1.191016	1.225043	1.259712	1.295029	1.331000
4	1.215506	1.262477	1.310796	1.360489	1.411582	1.464100
5	1.276282	1.338226	1.402552	1.469328	1.538624	1.610510
6	1.340096	1.418519	1.500730	1.586874	1.677100	1.771561
7	1.407100	1.503630	1.605782	1.713824	1.828040	1.948717
8	1.477455	1.593848	1.718186	1.850930	1.992563	2.143589
9	1.551328	1.689479	1.838459	1.999005	2.171893	2.357948
10	1.628895	1.790848	1.967151	2.158925	2.367364	2.593743
11	1.710339	1.898299	2.104852	2.331639	2.580426	2.853117
12	1.795856	2.012197	2.252192	2.518170	2.812665	3.138429
13	1.885649	2.132928	2.409845	2.719624	3.065805	3.452271
14	1.979932	2.260904	2.578534	2.937194	3.341727	3.7974[illegible]9
15	2.078928	2.396558	2.759032	3.172169	3.642483	4.177248
16	2.182875	2.540352	2.952164	3.425943	3.970306	4.594973
17	2.292018	2.692773	3.158815	3.700018	4.327633	5.054470
18	2.406619	2.854339	3.379932	3.996020	4.717120	5.559917
19	2.526950	3.025600	3.616528	4.315701	5.141661	6.115939
20	2.653298	3.207136	3.869685	4.660957	5.604411	6.727500
21	2.785963	3.399564	4.140562	5.033834	6.108808	7.400250
22	2.925261	3.603537	4.430402	5.436540	6.658600	8.140275
23	3.071524	3.819750	4.740530	5.871464	7.257875	8.954302
24	3.225100	4.048935	5.072367	6.341181	7.911083	9.849733
25	3.386354	4.291871	5.427433	6.848475	8.623081	10.834706
26	3.555673	4.549383	5.807353	7.396353	9.399158	11.918177
27	3.733456	4.822346	6.213868	7.988062	10.245082	13.109994
28	3.920129	5.111687	6.648838	8.627106	11.167140	14.420994
29	4.116136	5.418388	7.114257	9.317275	12.172182	15.863093
30	4.321942	5.743491	7.612255	10.062657	13.267679	17.449402
31	4.538040	6.088101	8.145113	10.867669	14.461770	19.194343
32	4.764942	6.453387	8.715271	11.737083	15.763329	21.113777
33	5.003189	6.840590	9.325340	12.676050	17.182028	23.225154
34	5.253348	7.251025	9.978114	13.690134	18.728411	25.547670
35	5.516015	7.686087	10.676582	14.785344	20.413968	28.102437

EXAM. If a boy, at birth, have a legacy of $5000 left him, how much will he have to receive at the age of 21, at the rate of 4% compound interest?

The amount of $1, taken from the table, for 21 years at 4% is 2.278768, and 2.278768 × 5000 = $11393.84, the required amount.

RULE II. *To find the principal, which, at a given rate and in a given time, would amount to a given sum; Or, to find the present worth of a sum at compound interest, for a given time and at a given rate: Divide the given sum by the amount of* $1 *for the given time and rate.*

EXAM. What sum must be invested at the birth of a child, at 4% per annum, compound interest, so as to amount to $11393.84 at the end of 21 years?

Here, the amount of $1 for 21 years, at 4%, taken from the table is 2.278768, and dividing the given sum by this, we have $11393.84 ÷ 2.278768 = $5000.

RULE III. *To find the time in which a given principal will produce a given amount. compound interest, at a given rate: Divide the given amount by the given principal; the result is the amount of* $1 *for the time required. If this amount be found in the table, the required time will be found in the column marked years, opposite said amount.*

EXAM. In what time will $5000 amount to $11393.84, at 4%, compound interest?

Here, we have $11393.84 ÷ 5000 = 2.278768, the amount of $1 for the required time. This number, 2.278768 is found in the table in the 4% column, and opposite is found 21 years, the time required.

If, when the amount is divided by the principal, the result is not found in the table, proceed as illustrated in the following

EXAM. In what time will $5000 amount to $10000, compound interest, at 6%, in other words, how long will it take to have money double itself, at 6%, compound interest?

Here, dividing $10000 by $5000, we obtain $2, the amount of $1 for the required time. We now take from the table the nearest amount in excess of $2. Looking along the 6% column we find 2.012197, and looking in the yearly column opposite, we find 12 years, or the time in which $1 would amount to 2.012197, the difference being .012197, the interest which must accrue in the fraction of a year on $2.

Now, the interest on \$2 for 1 year at 6% is 12¢, and the question now is, if the interest on \$2 for 1 year be .12, in what time will \$2 give .012197 interest? For this we have the following proportion: As .12 : .012197 :: 1 yr. x or the time obtained by dividing .012197 by .12 = .1016 decimal of a year = 1 mo. 6 da., which is deducted from 12 years as found in the table; the required time is 11 yrs. 10 mo. 24 da. (12 yrs. — 1 mo. 6 da.).

A more expeditious method for finding the time when a sum of money will double itself, at any rate per cent., compound interest, is as follows:

RULE. *Divide the number* 69.3 *by the rate per cent. and to the result add decimal* .35; *the result is the time in years and the decimal of a year.*

Thus, taking the foregoing example, to find in what time \$5,000 *will double itself: Dividing* 69.3 *by* 6 *we get* 11.55 *to which decimal* .35 *is added; the result is* 11.90 *years. Reducing decimal* .90 *to months and days, we get* 11 *yrs.* 10 *mo.* 24 *days as found above.*

RULE. *To ascertain the sum of money necessary to be laid aside at the end of each year, to provide for the payment of a given amount, in a given number of years, at a stated rate of interest compounded annually: Divide the interest for one year upon the given sum by the compound interest of* \$1 *at the given rate, and given number of years; the result will be the annual sum.*

EXAM. What sum must be put into the sinking fund at the end of each year at 4 per cent. compound interest, to provide for the payment of \$20000, payable in 10 years?

Here, the interest of \$20000 for one year at 4% = \$800; and the amount of \$1 for ten years at 4% compound interest (taken from comp. int. table, page 298) is 1.480244. Deducting \$1 from this amount, the result is the compound interest, .480244.

Applying the rule, now, we have: \$800 ÷ .480244 = \$1665.82, the required sum.

To prove that the work is correct, add to \$1665.82 its interest for a year at 4%, and to the result add the sum to be laid aside (\$1665.82) make the sum a new principal, to which add its interest for a year at 4%, and to the sum add \$1665.82 for the next principal. Repeat the process until you have found \$20000. The work is left for the student.

If the money is to be laid aside at the *beginning* of each year; divide the result found above by the amount of \$1 for one year at the given rate; thus, \$1665.82 ÷ 1.04 = \$1601.75, the sum required to be laid aside at the beginning of the year.

BOND COMPUTATIONS.

RULE. *To find the present value of Bonds:* (1) *Divide* \$1 *by its amount at compound interest for the given time and at the desired rate.* (2) *Subtract the result thus found from* \$1, *and multiply the remainder by the difference of rates.* (3) *Divide this last result by the desired rate; this will give either a premium or a discount.*

If the desired rate be greater than the bond rate, it will be a discount to be subtracted from \$1; *and if less, it will be a premium to be added; the result, in either case, will be the value of* \$1, *and* 100 *times this the value of the bond.*

EXAM. 1. What must be paid for a bond maturing in 10 years with 4% interest, payable annually, to pay the purchaser 6%.

The amount of \$1 @ 6% compound interest for 10 years = 1.790848. (Taken from comp. interest table, page 298.)

Now, 1 ÷ 1.790848 = .558395; next, 1 — .558395 (1.000000 — .558395) = .441605 then, .441605 × 2 (6%—4%) = .883210; and .883210 ÷ 6 = .147202, which is a discount to be subtracted from \$1, thus, 1 — .147202 = .852798; this last is the value of \$1 on the proposed terms; and 100 times that = \$85.2798, the value of the bond.

EXAM. 2. If the interest were paid semi-annually, instead of annually, in the foregoing, what would be the value of the bond?

Referring to the compound interest table page 298, we find the amount of \$1 for 20 years @ 3% to be 1.806111.

Now, 1 ÷ 1.806111 = .553676; and 1 — .553676 = .446324; next, .446324 × 2 = .892648 then, .892648 ÷ 6 = .148775, which is to be subtracted from \$1, thus, 1 — .148775 = .851225, and 100 times this = \$85.1225.

EXAM. 3. What must be paid for bonds amounting to \$10000 maturing in 10 years, with interest @ 6%, to net 4%, interest payable semi-annually?

Here, the amount of \$1 for 20 years @ 2% (taken from the comp. int. table, page 298) is found to be 1.485947.

Now, 1 ÷ 1.485947 = .672971; next, 1 — .672971 = .327029; and .327029 × 2 = .654058. Dividing this by the net rate, we have .654058 ÷ 4 = .163514, which is a premium to be added to \$1, thus, 1 + .163514 = 1.163514, the value of \$1 at the proposed terms; and 10000 times 1.163514 = \$11635.14, the value of the bonds.

EXAM. 4. What must be paid for a 5% bond maturing in 4 years, to net 4%, interest payable annually?

In this, the amount of $1 taken from the int. table for 4 years @ 4% = 1.169859. And 1 ÷ 1.169859 = .854804: now, 1 — .854804 = .145196 (the difference of rates being 1, there is no need to multiply) so we divide this by the required net rate, thus, .145196 ÷ 4 = .036299 which is a premium to be added giving 1.036299, the value of $1. and 100 times that = $103.6299 or $103.63 the cost of the bond.

To prove this to be correct, add to $103.63 its interest for a year at 4% = $4.14, the amount is $107.77, from which deduct $5 (bond int.) the result is $102.77. Compute a year's interest on this principal and deduct the bond interest; repeat the process until at maturity the value is found to be $100.

ANNUITIES.

RULE. *To find the amount of an annuity, payable yearly, the payments of which are forborne for a given time, compound interest being charged on them as they became due.*

(1) *Divide the compound interest of* $1 *for the given time and rate, by the rate per cent.; the result will be the amount of an annuity of* $1 *forborne for the proposed time.*

(2) *Multiply this annuity by the given annuity; the result will be the amount required.*

EXAM. If a person save $500 a year, and improve it at 4% per annum, compound interest; how much will he be worth at the end of 21 years?

In this, the amount of $1 for 21 years, at 4% compound interest is 2.278768. And deducting $1 from this we have the compound interest of $1 = 1.278768.

Applying the rule, we have 1.278768 ÷ .04 × 500 = $15984.60, what the person is worth at the end of 21 years.

PROOF: What sum must be laid aside at the end of each year, for 21 years, at 4% compound interest, to amouut to $15984.60?

Applying the rule for the sinking fund, given on page 301, we find the interest of $15984.60 for 1 year at 4% = $639.38, and dividing this by the compound interest of $1 we have: $639.38 ÷ 1.278768 = $500.

To Discharge a Given Debt in Several Equal Payments, Including Principal and Interest, at Any Rate Per Cent.

Rule. (1) *Multiply the amount of the debt at compound interest for the given time and rate, by the rate per cent.* (2) *Divide the result by the compound interest of* \$1 *for the given time and rate; the result will be the payment.*

Exam. A manufacturer sold 5 pianos at \$720 each, agreeing to take in settlement six *equal half yearly payments*, including principal and interest, at 6% per annum; what was the amount of each payment?

Solution.— \$720 × 5 = \$3600, the total cost.

The yearly rate being 6%, the rate half yearly is 3%, and there are to be 6 equal payments. Now, the amount of \$1 for the first interval is \$1.03, which is to be raised to the 6th power; thus, $1.03^6 = 1.03 \times 1.03 \times 1.03 \times 1.03 \times 1.03 \times 1.03 = 1.194052$, which is the amount of \$1 at compound interest for the given time and rate; and deducting \$1 from this amount, gives the compound interest .194052, on \$1 for the given time and rate.

Applying the rule, now, we have: $\frac{1.194052 \times 3600 \times .03}{.194052} = \$664.55$, the amount of each payment.

Note.—The contracted method for the multiplication of decimals given on pages 293 and 294 can be used in raising 1.03 to the 6th power. Or, the amount of \$1 for 6 years at 3% may be had at a glance from the compound interest table, pages 298 and 299.

Such problems as the foregoing, however, can be more easily and rapidly solved by the use of the table given on pages 306 and 307, where the amount required to discharge the debt of \$1, in equal payments, is given.

To illustrate, let us take the foregoing example. Referring to the table, we look for 6 years, and opposite, under 3%, is the decimal .184598. This is the amount required to discharge \$1, in equal payments, in 6 years, at 3%; and multiplying this by the given debt, we obtain the amount of each equal payment; thus, .184598 × 3600 = \$664.55, as found by the rule.

In connection with the foregoing rule and example we subjoin the following:

PROOF.

	$3600.	Brought forward	$1879.75
Int. on $3600 for 6 mo. at 3% =	108	Int. on $1879.75; 6 mo. 3% =	56.39
	$3708		$1936.14
Amt. of 1st payment =	664.55	Amt. of 4th payment =	664.55
	$3043.45		$1271.59
Int. on $3043.45; 6 mo. 3% =	91.30	Int. on $1271.59; 6 mo. 3% =	38.15
	$3134.75		$1309.74
Amt. of 2d payment =	664.55	Amt. of 5th payment =	664.55
	$2470.20		$645.19
Int. on $2470.20; 6 mo. 3% =	74.10	Int. on $645.19; 6 mo. 3% =	19.36
	$2544.30		$664.55
Amt. of 3d payment =	664.55	Amt. of 6th or last payment =	664.55
Carried forward	$1879.75		

As a further illustration of the rule, we give the following:

EXAM. To settle a debt of $6,000, three notes, each for an equal sum, and drawn at 4 months, are given; what is the amount of each note, including principal and interest, at 6% per annum?

Solution. — The annual rate being 6%, the rate for 4 mo. is 2%, and there are to be 3 equal payments.

The amount of $1 for the first interval of 4 mo. at 2% (.02) is 1.02; and this raised to the 3d. power gives the amount of $1 for the proposed time and rate; thus, $1.02 \times 1.02 \times 1.02 = 1.061208$; and deducting $1 from this amount gives the compound interest of $1, viz. .061208.

Applying the rule, now we have: $\dfrac{1.061208 \times 6{,}000 \times .02}{.061208} = \$2{,}080.528$ the amount of each note.

Now, by referring to the table on the next page, and looking in the yearly column for 3 years (3 payments) and opposite, in the column for 2% we find decimal .346755; multiplying this by the debt, we get the amount of each note, including principal and interest; thus $.346755 \times 6{,}000 = \$2{,}080.53$. (The proof is left for the student.)

Solve the following problem by using the table:

A debt of $50,000 is to be paid in 4 years, in 8 equal semi-annual payments; what is the amount of each payment, including principal and interest, at 5% per annum?

Turning to the table, we look for 8 years (8 payments) and opposite, in the $2\frac{1}{2}$% column (half of 5%), we find decimal .139467, and multiplying this by the debt, we have $.139467 \times 50{,}000 = \$6{,}973.35$, the required amount.

Table showing the amount required to discharge a debt of $1 in equal payments, from 2 to 21 years, at the rates given.

Years.	2%.	2½%.	3%.	3½%.	4%.	Years.
2	.515049	.518827	.522611	.526400	.530196	2
3	.346755	.350135	.353530	.356930	.360348	3
4	.262625	.265818	.269022	.272250	.275494	4
5	.212160	.215247	.218355	.221482	.224627	5
6	.178526	.181550	.184598	.187668	.190762	6
7	.154512	.157495	.160506	.163544	.166609	7
8	.136510	.139467	.142456	.145476	.148528	8
9	.122515	.125453	.128106	.131446	.134493	9
10	.111327	.114258	.117231	.120241	.123291	10
11	.102178	.105106	.108077	.111092	.114149	11
12	.094559	.974871	.100462	.103484	.106552	12
13	.088118	.910483	.094030	.097062	.100142	13
14	.082602	.860208	.088526	.091571	.094669	14
15	.077826	.807656	.083767	.086825	.088941	15
16	.073650	.765989	.079611	.082685	.085820	16
17	.069970	.729278	.075953	.079043	.082198	17
18	.066702	.696701	.072709	.075871	.078993	18
19	.063782	.667606	.069814	.072940	.076138	19
20	.061157	.641471	.067216	.070361	073582	20
21	.058785	.617872	.064872	.068037	.071280	21

As an aid to the solution of problems of the foregoing nature, recourse can be had to the Compound Interest Tables given on pages 298 and 299.

EXAM. Suppose it were required to pay off a debt of $40000 in two years, in 8 equal quarterly payments, including principal and interest, at 8% per annum; what should be the amount of each payment?

Solution.— The yearly rate being 8%, the quarterly rate is 2% (.02) and there are to be 8 payments. The amount of $1 for the first interval is $1 plus .02 = 1.02; and for 8 intervals, the amount of $1 is 1.02^8; 1.02 raised to the 8th power, requiring seven multiplications to get the amount. To avoid this labor, we refer to the interest table, and looking in the yearly column for 8 years (8 payments) and opposite, in the 2% rate column, we find 1.171659, the amount of $1

Table showing the amount required to discharge a debt of $1 in equal payments, from 2 to 21 years, at the rates given.

Years.	4½%	5%.	6%.	7%.	8%.	Years.
2	.533998	.537805	.545437	.553091	.560769	2
3	.363774	.367208	.374109	.381051	.388034	3
4	.278743	.282012	.288591	.295228	.301921	4
5	.227792	.230974	.237396	.243890	.250457	5
6	.193878	.197017	.203363	.209796	.216315	6
7	.169701	.172819	.179135	.185553	.192072	7
8	.151610	.154722	.161036	.167607	.174015	8
9	.137572	.140690	.147022	.153486	.160079	9
10	.126321	.129506	135881	.147547	.149029	10
11	.117248	.120389	.126793	.133357	.140076	11
12	.109666	.112825	.119278	.125902	.132695	12
13	.103275	.106456	.112960	.119651	.126522	13
14	.097820	.101024	.107759	.114345	.121297	14
15	.093114	.096343	.102963	.109795	.112226	15
16	.089015	.092269	.098952	.105857	.112098	16
17	.085418	.088699	.095445	.102425	.109629	17
18	.082237	.085546	.092356	.099413	.106702	18
19	,079407	.082090	.089621	.096753	.104128	19
20	.076876	.080243	.087185	.094393	.101852	20
21	.074601	.077996	.085005	.092286	.099832	21

for the proposed time and rate; and deducting $1 from this amount, we get .171659, the compound interest.

Now, by the rule on page 304, we have $\frac{1.171659 \times 40000 \times .02}{.171659} =$ $5460.42, amount of each payment.

The problem can be more readily solved, however, by using the Debt Table, as follows:

Looking in the yearly column for eight years (8 payments) and opposite in the 2% rate column, we find decimal .136510, the amount required to discharge the debt of $1 in the proposed time at the given rate; and this multiplied by the debt gives the payment, at once; thus, .136510 × 40000 = $5460.40.

NOTE. — There is a discrepancy of 2¢ in the answer given above; but if the decimal in the interest table were carried out to eight places, 17116938, instead of six, the answer would be $5460.39 + a little in excess of 39¢. Six places of decimals, however, are sufficient for all practical purposes.

THE CUBE ROOT.

RULE I.— *Point off the given number into periods of three figures each, beginning at the units.*

II. *Find the greatest cube of the left-hand period, and write its root as the first figure of the required root.*

III. *On the left of the dividend write three times the square of the first figure of the root, and annex two ciphers, or dots to represent them; this number is trial divisor by which find the next figure of the root.*

IV. *On the left of the trial divisor, and one line lower, write the root figure thus found in the place of units, and three times the part of the root formerly found write in the place of tens, hundreds, etc.*

V. *Multiply this last number by the last figure of the root, set the result under the trial divisor, and add the two for the complete divisor.*

VI. *Multiply this complete divisor by the quotient figure, subtract and bring down another period.*

VII. *Lastly, underneath the complete divisor, write the square of the last figure found in the root; add this square and the two numbers immediately above it; annex two ciphers (dots) and the result will be another trial divisor, with which proceed as before*

NOTE.—If a cipher occur in the root, annex two more ciphers (dots) to the trial divisor, and another period to the dividend; then proceed as before.

EXAM. Extract the cube root of 389816897625.

			389816897625(7305
7	49		343
	3		46816
	147··	t. d.	46017
213	639		799897625
	15339	c. d.	799897625
	9		
	15987····	t. d.	
	109525		
21905	159979525	c. d.	

Having proceeded as far as the second figure (3) of the root, we add its square (9) to the *two* numbers (639 and 15339) immediately above it, getting 15987 to which we annex two ciphers, or dots to represent them, and we have 1589700 (15897··) for trial divisor (t. d.). We next bring down another period (897) to the dividend and we have 799897. The trial divisor is not contained in this number, so we set a cipher in the root, annex two more ciphers (dots) to the trial divisor, and another period (625) is brought down to the dividend.

The trial divisor (t. d.) is now 158970000 (15897····) by which we find the next figure (5) in the root. Setting 5, now, to the left of the trial divisor, and one line lower, in the place of units, and 5 times 730, the part of the root formerly found, in the place of tens, hundreds, etc., we get 21905.

Finally, 5 times 21905, or 109525 is added to the trial divisor which gives 159979525 for the complete divisor (c. d.); and 7305 is the required root.

EXAM. Required the cube root of 477 to four places of decimals.

7	49		477 (7.8133
	3		343
	147··	t. d.	134000
218	1744		131552
	16444	c. d.	2448000
	64		1827541
	18252··	t. d.	620459000
2341	2341		549175797
	1827541	c. d.	71283203000
	1		54940781637
	1829883··	t. d.	
23433	70299		
	183058599	c. d.	
	9		
	183128907··	t. d.	
234393	703179		
	18313593879		

Having found 7, the first figure of the root, 3 times its square, 147, with two ciphers (dots) annexed, is trial divisor. Annexing three ciphers (a period) to the dividend, we find the next figure, 8, of the root, inserting the decimal point. Then 8 is set to the left of the trial divisor, and one line lower, in the place of units, and three times 7, the first root figure, or 21, is set in the place of tens and hundreds, giving 218. This is now multiplied by 8, and the result, 1744, is added to the trial divisor, giving 16444 for the complete divisor. 8 times this is now set under the dividend, and the result, 131552, subtracted. Finally, the square of 8 = 64, is set under the complete divisor and added to the *two* numbers immediately above it (included in the brace), and the sum of the three numbers with two ciphers (dots) annexed, is the next trial divisor with which find the next figure of the root, and proceed as before.